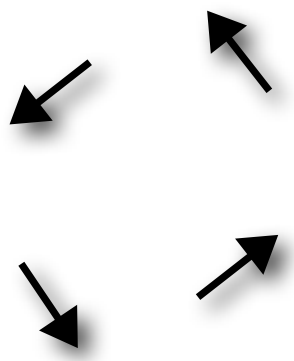
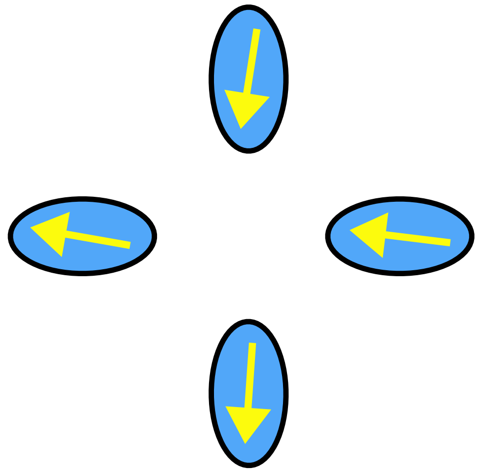
Metastability in magnetic island lattices

Nov. 17, 2023



Gary Wysin
Kansas State University
Manhattan, Kansas, U.S.A.

wysin@phys.ksu.edu
www.phys.ksu.edu/personal/wysin



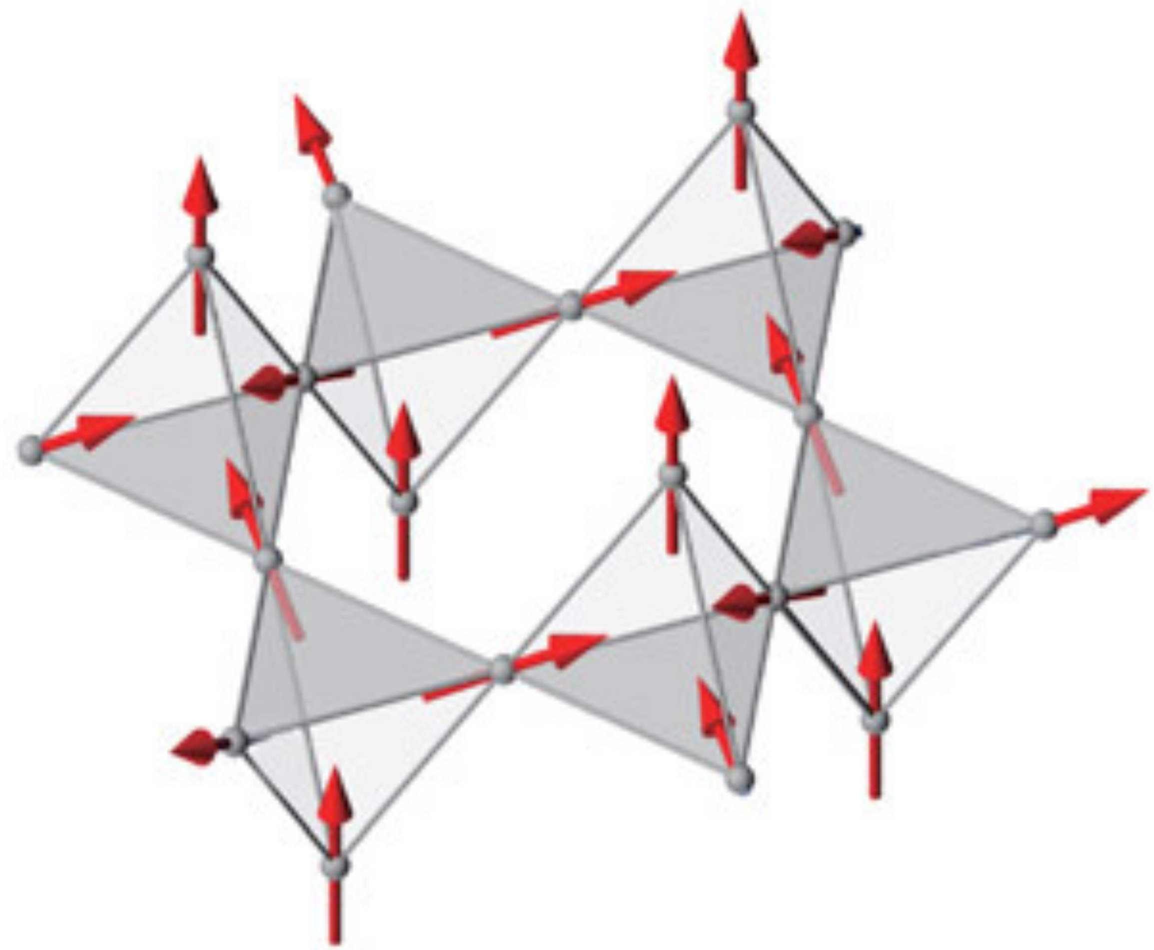
Outline

1. What are magnetic island lattices?
2. What properties can be studied?
3. How do their magnetic dipoles interact?
4. Ground state vs. excited states and metastable states.
5. A 1D model vs. a 2D model.
6. Possible technological applications.

A *real* 3D spin-ice.

A rare-earth
pyrochlore compound.

Pr spins at
corners of tetrahedrons.

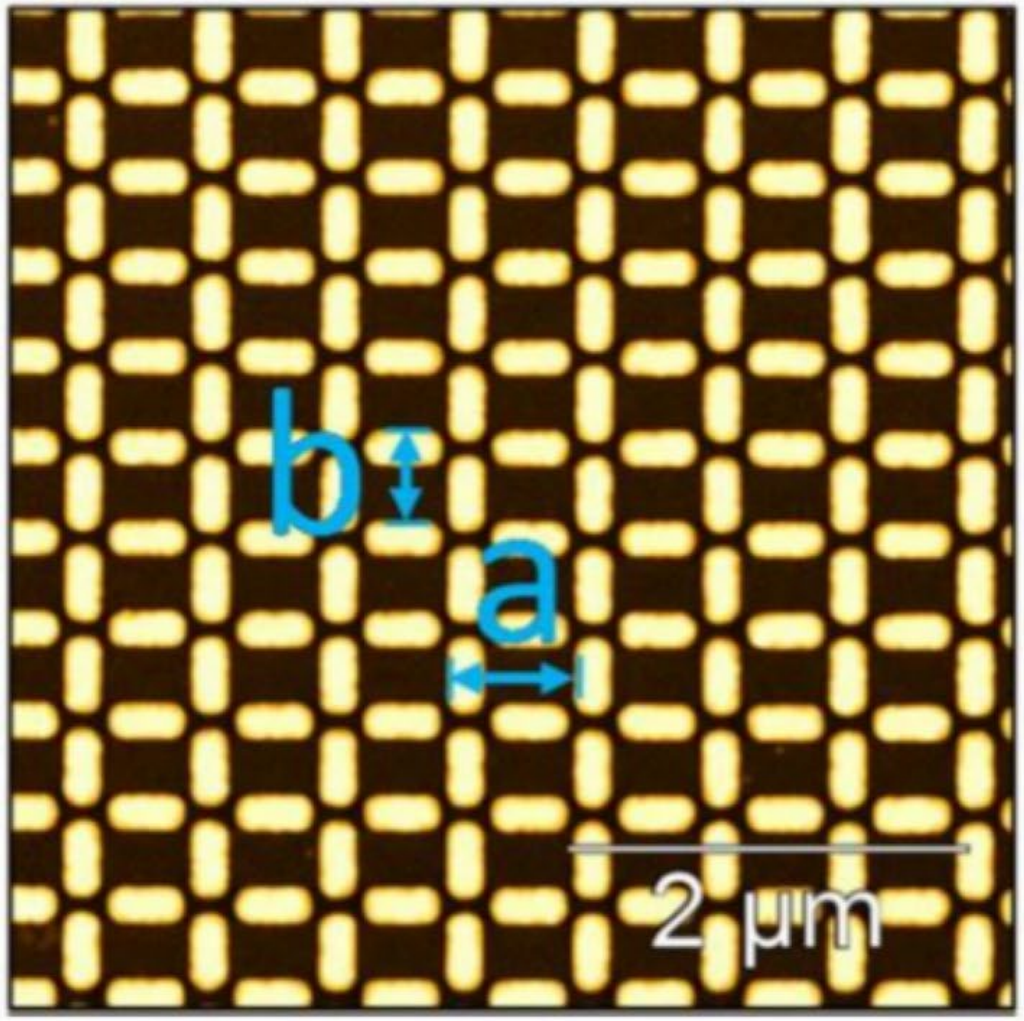


The interesting properties of $Pr_2Ir_2O_7$ are rooted in its crystal structure, called a pyrochlore lattice: four praseodymium (Pr) ions, each of which carries a magnetic 'spin', form a tetrahedral cage around an oxygen (O) ion. At low temperatures, the spins of materials with this structure often 'freeze' into what is called a 'spin ice' (Fig. 1) because of its similarity to the way hydrogen ions form around oxygen in water ice. (phys.org/news/)

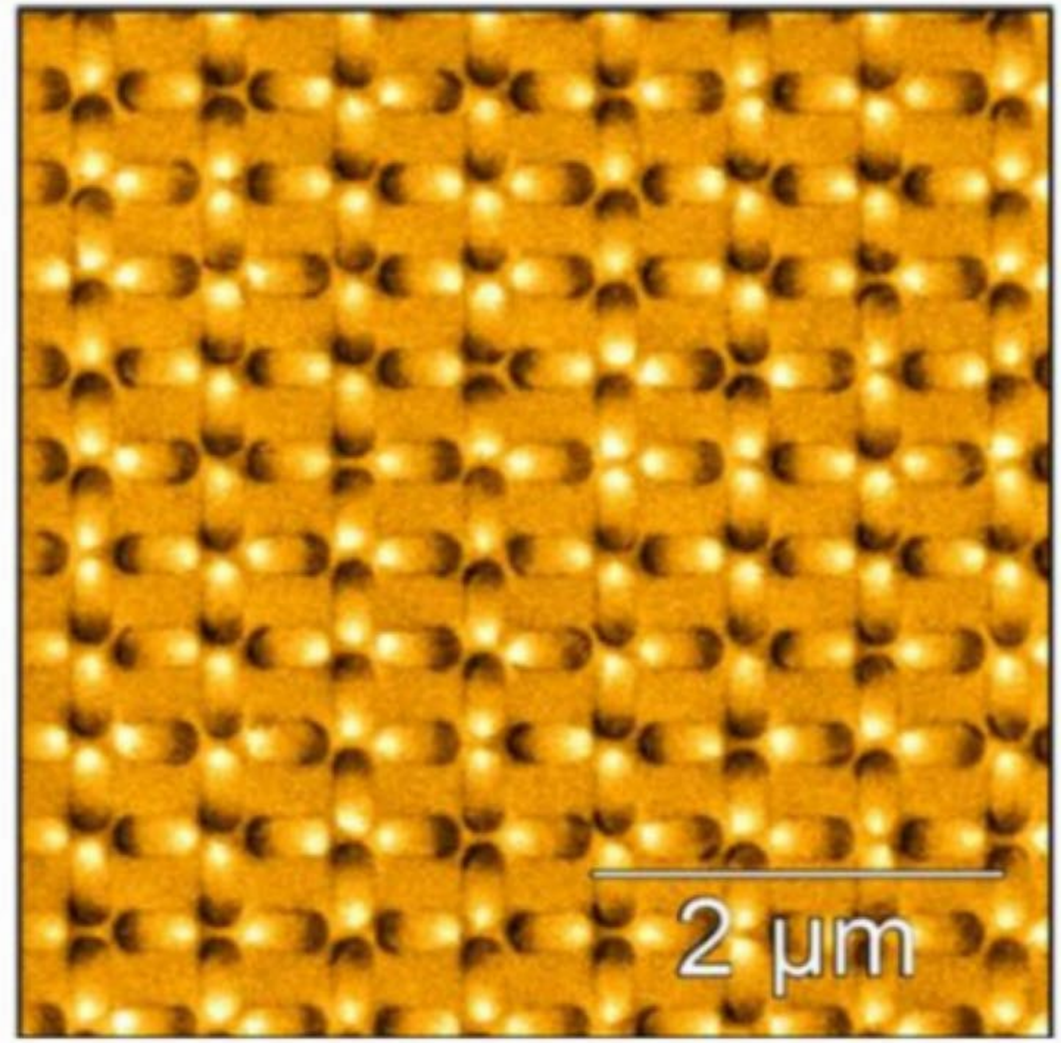
Realization of Rectangular Artificial Spin Ice and Direct Observation of High Energy Topology

I. R. B. Ribeiro[1,6], F. S. Nascimento[2], S. O. Ferreira[1], W. A. Moura-Melo[1], C. A. R. Costa[3], J. Borme[4], P. P. Freitas[4], G. M. Wysin[5], C. I. L. de Araujo[1] & A. R. Pereira[1]

Artificial 2D spin-ice. Arrays of elongated magnetic islands, dominated by anisotropy & dipole-dipole interactions.



Atomic force microscope topography, 300 x 100 x 20 nm islands.



Magnetic force microscope image showing N (bright) and S (dark) poles.

Artificial spin ice mimics the behavior of 3D spin ices of rare earths in lattice of corner sharing tetrahedra of a pyrochlore structure.

in Review article: **Advances in artificial spin ice,**
Sandra Skjærvø et al. Nat. Rev. Phys. 11/08/19:

Artificial spin ices are metamaterials made up of coupled nanomagnets arranged on different lattices that exhibit a number of interesting phenomena, such as emergent magnetic monopoles, collective dynamics and phase transitions.

Signatures of the magnetic configurations are given by the specific spin-wave resonances in artificial spin ice, which offer a platform for programmable spin-wave devices, in particular magnonic crystals.

The established artificial spin ices are arranged on square and kagome lattices. New geometries include both periodic and aperiodic, different magnet shapes and anisotropies, and 3D structures.

Future work involves the development of applications including computation, data storage, encryption and reconfigurable microwave circuits.

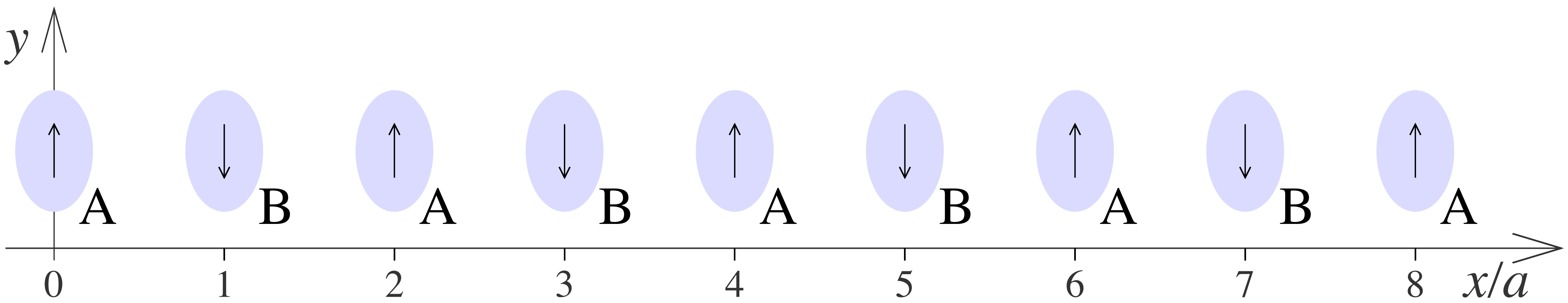
Another aspect:

Easily generated metastable states above the ground state, that might be manipulated by outside control forces.

Metastable states could be useful as detectors or controllable oscillators.

Their oscillation frequencies can change rapidly vs some parameter when at a critical value of that parameter.

A 1D island chain with metastability

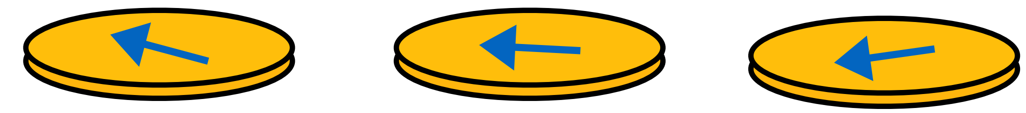


Thin elliptical islands.

2 sublattices, A,B.

They have shape anisotropy and dipole-dipole interactions.

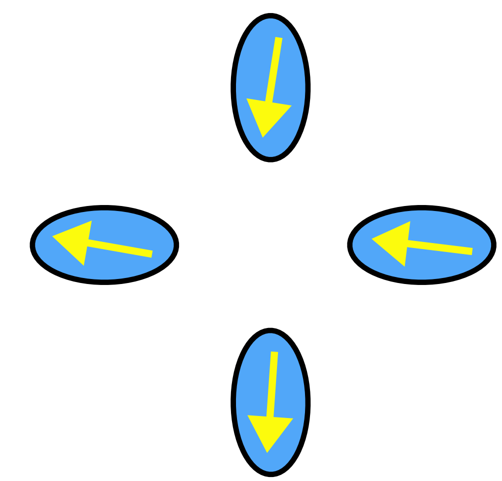
Magnetic Nano-Islands
(elements of artificial spin-ice)



Approx. 50 nm - 5 μm wide but only 10 nm thick.
Individual & in arrays, high-permeability soft magnetic materials.
Grown with techniques of epitaxy & lithography on a non-magnetic substrate.
Form arrays of particles that can interact with each other or applied fields.

Primary physics effects -

- magnetostatics controlled by island geometry.
- discrete energy states for data storage.
- spintronics controlled by current injection.
- magnetic oscillators controlled by applied fields.
- frustration in ordered arrays of islands (artificial spin-ice).



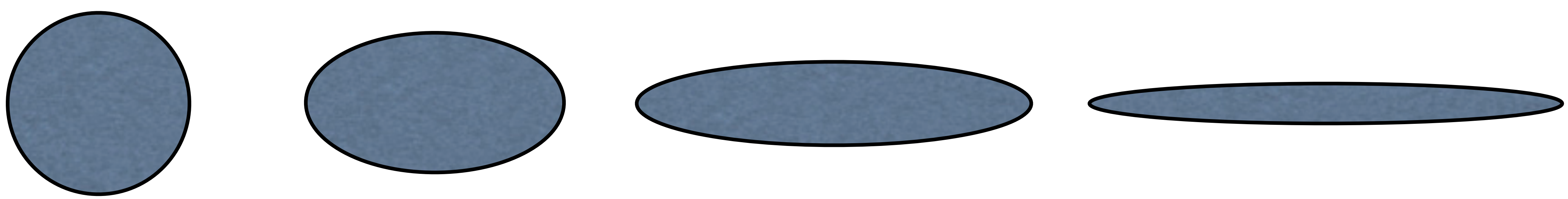
Several principle states of a nano-island:
(1) quasi-single domain; (2) vortex; (3) multi-domains & domain walls.

~ increasing size ~

Typical magnetic island features:

1) Vortices. The static and dynamic properties of single vortices that behave as particles with charges (==> micro-oscillators).

2) Magnetostatic anisotropy of the islands themselves. Also known as shape anisotropy because it depends mostly on the surfaces.

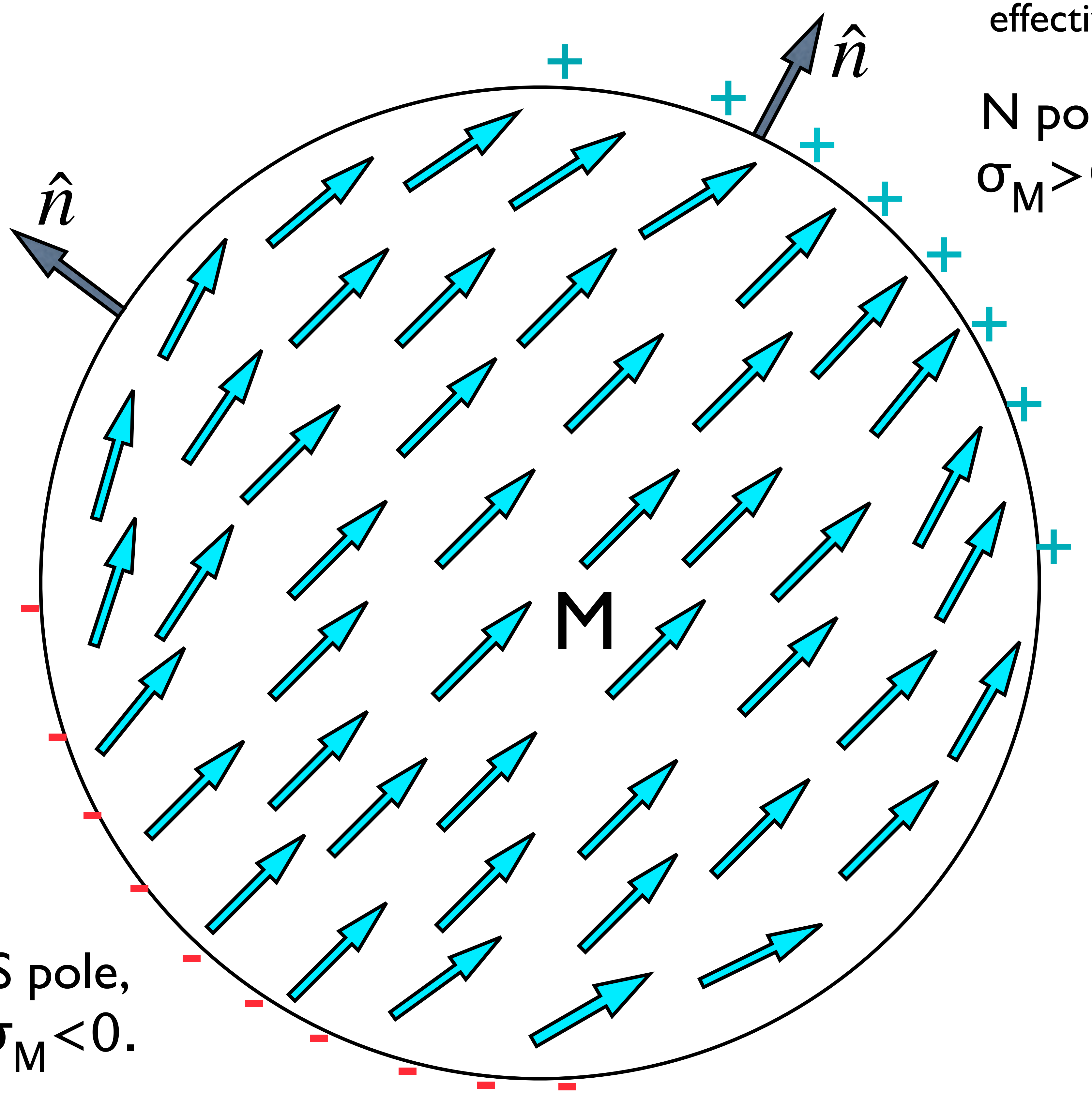


isotropic elliptic Ising-like

3) Spin-ices, frustration. Especially for elongated islands with Ising-like states, interactions within their arrays, that lead to frustrated statics and dynamics.

Quasi-single-domain state.

Magnetization M determines an effective surface charge density:



N pole, $\sigma_M > 0$.

$$\sigma_M = \vec{M} \cdot \hat{n},$$

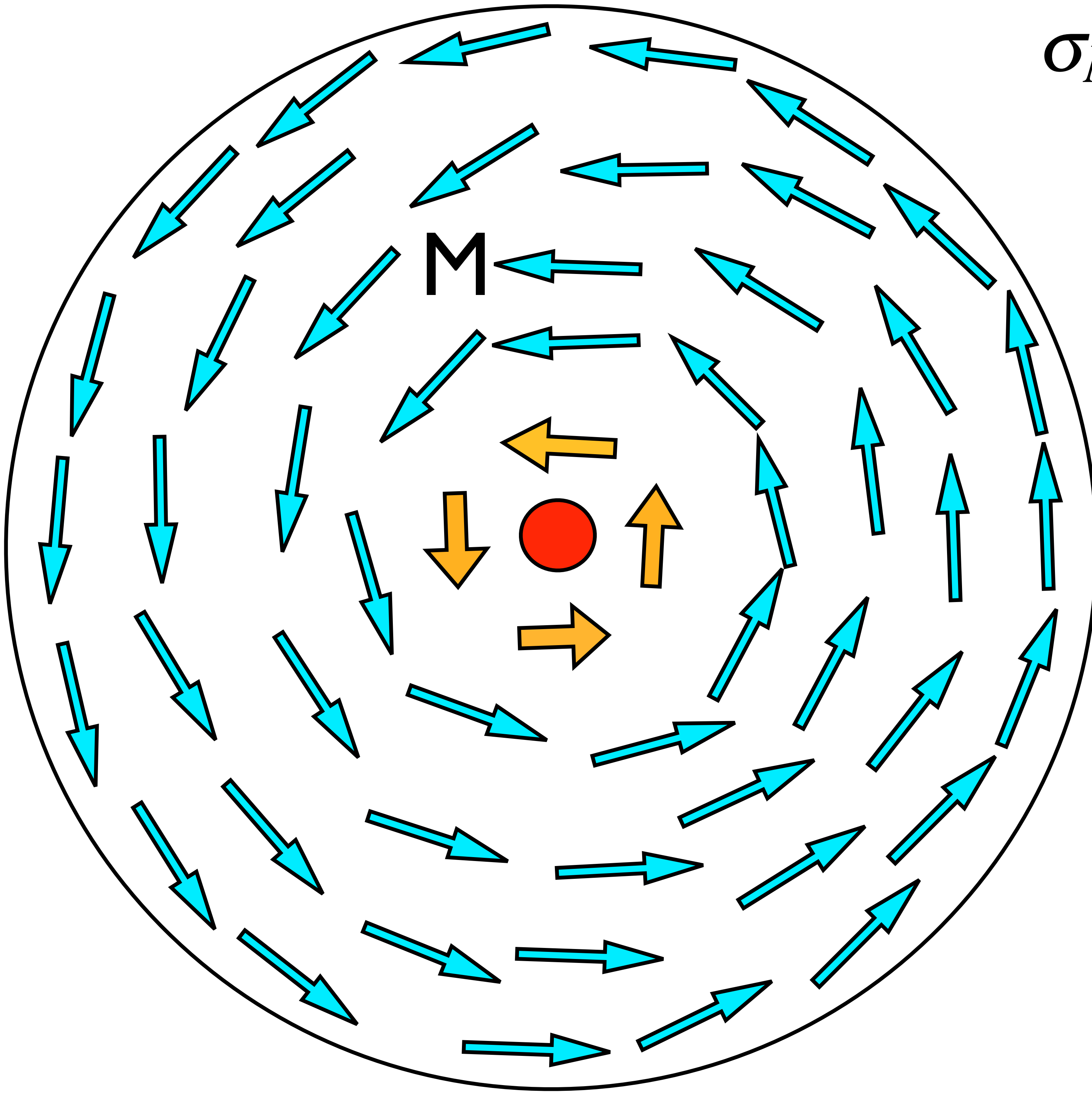
S pole, $\sigma_M < 0$.

The poles produce large stray-field energy.

But ferromagnetic exchange energy is small.

Vortex state

Very little magnetic surface charge density.
Stable only above a minimum radius



$$\sigma_M = \vec{M} \cdot \hat{n},$$

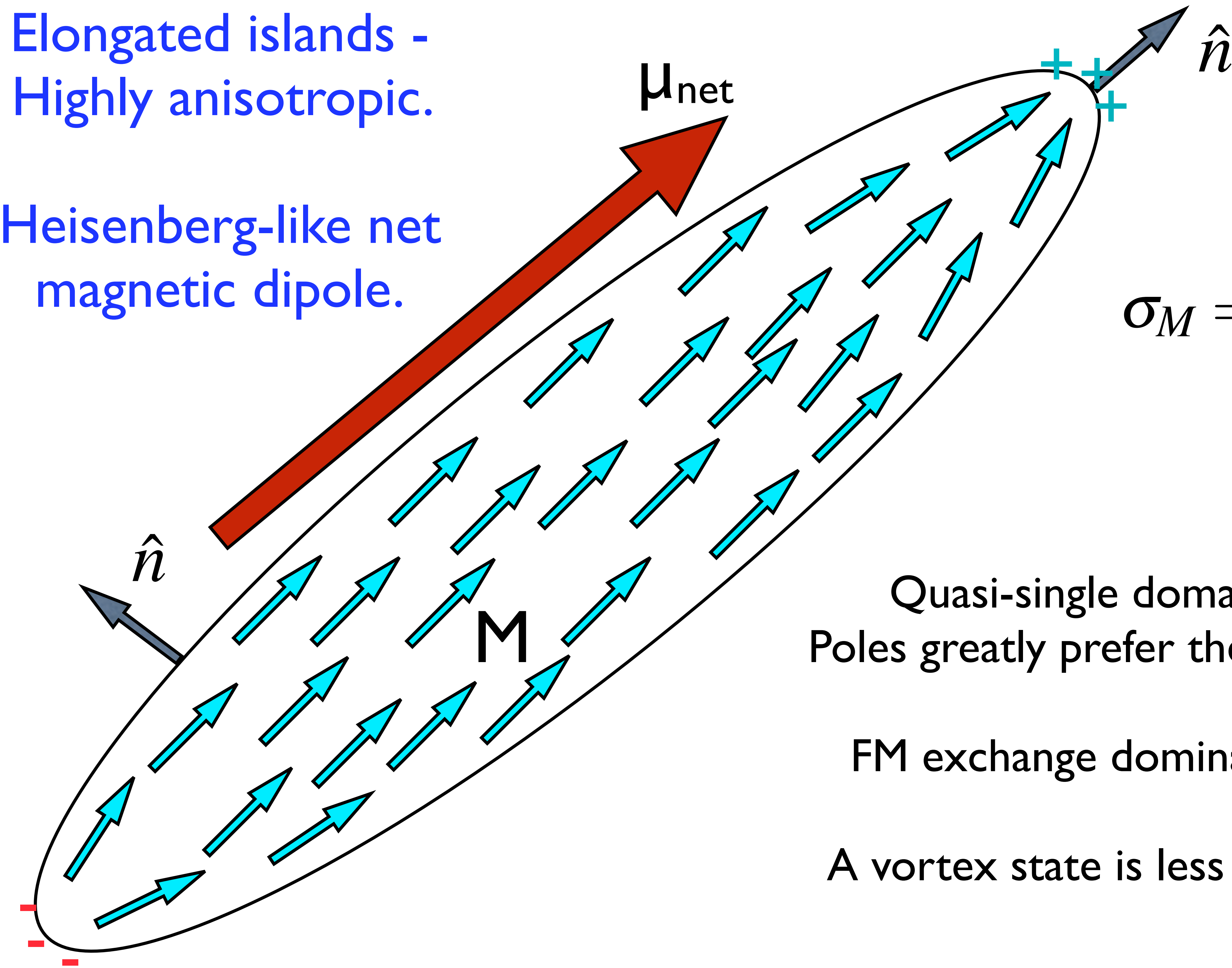
Has small poles ($\sigma_M = \pm M_z$) only in the core.

The stray-field energy is small.

But the ferromagnetic exchange energy is large.

Elongated islands -
Highly anisotropic.

Heisenberg-like net
magnetic dipole.



$$\sigma_M = \vec{M} \cdot \hat{n},$$

Quasi-single domain.
Poles greatly prefer the ends.

FM exchange dominates.

A vortex state is less likely.

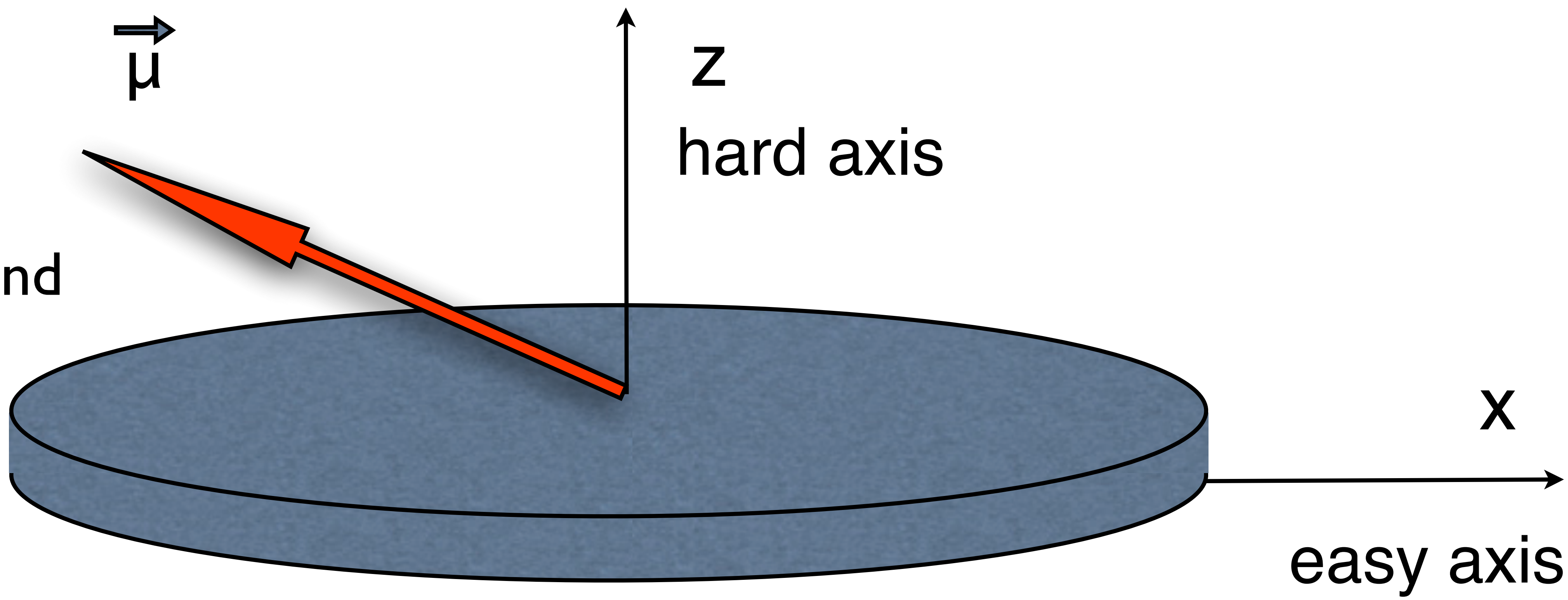
Model for magnetic anisotropy of elliptical islands.

Total magnetic dipole moment = $\vec{\mu}$. Single domain is assumed and $\vec{\mu}$ has a fixed magnitude.

$$E = E_0 + K_1\left[1 - (\hat{\mu}\cdot\hat{x})^2\right] + K_3(\hat{\mu}\cdot\hat{z})^2$$

Include also applied field energy: $-\mu H_{ext}$

$L_x \times L_y \times L_z$ island



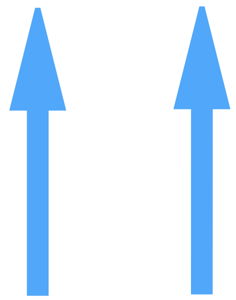
μ direction = (ϕ_n, θ_n)

angle in *xy*-plane

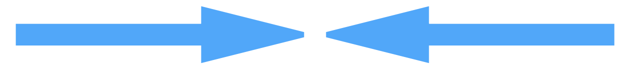
angle from xy-plane

dipolar interactions on a 1D island chain

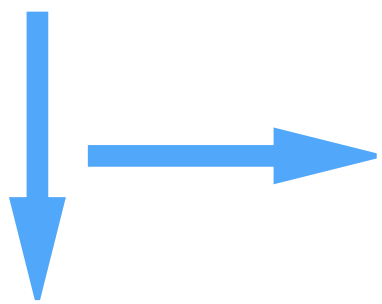
$$\mathcal{H} = -\frac{\mu_0}{4\pi}\frac{\mu^2}{a^3}\sum_{i>j}\frac{[3(\hat{\mu}_i\cdot\hat{r}_{ij})(\hat{\mu}_j\cdot\hat{r}_{ij}) - \hat{\mu}_i\cdot\hat{\mu}_j]}{(r_{ij}/a)^3} \qquad D = \frac{\mu_0}{4\pi}\frac{\mu^2}{a^3}$$



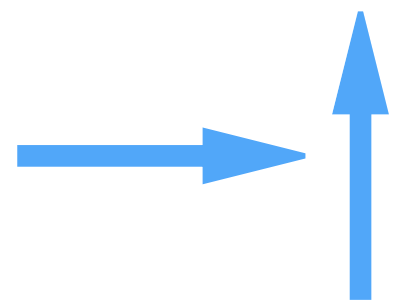
high energy (D)



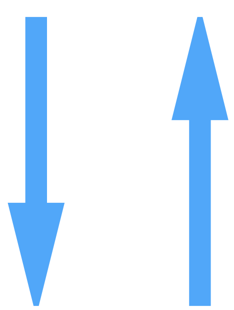
highest energy (2D)



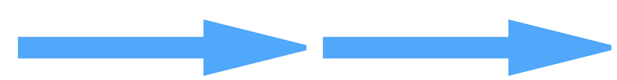
medium energy (0)



medium energy (0)



low energy (-D)



lowest energy (-2D)

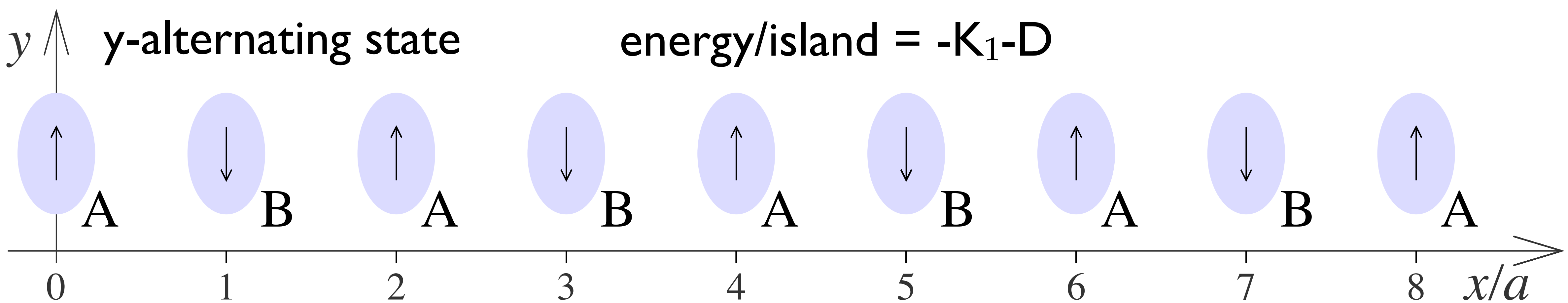
Interactions = dipolar + shape anisotropy + external field

$$H = \sum_{n=1}^{N} \{ D\left[\mathbf{S}_n \cdot \mathbf{S}_{n+1} - 3(\mathbf{S}_n \cdot \hat{x})(\mathbf{S}_{n+1} \cdot \hat{x})\right] - K_1 \left(S_n^y\right)^2 + K_3 \left(S_n^z\right)^2 \}$$

easy axis hard axis

energy scale $D = \frac{\mu_0}{4\pi}\frac{\mu^2}{a^3}$

y-alternating state energy/island = $-K_1 - D$

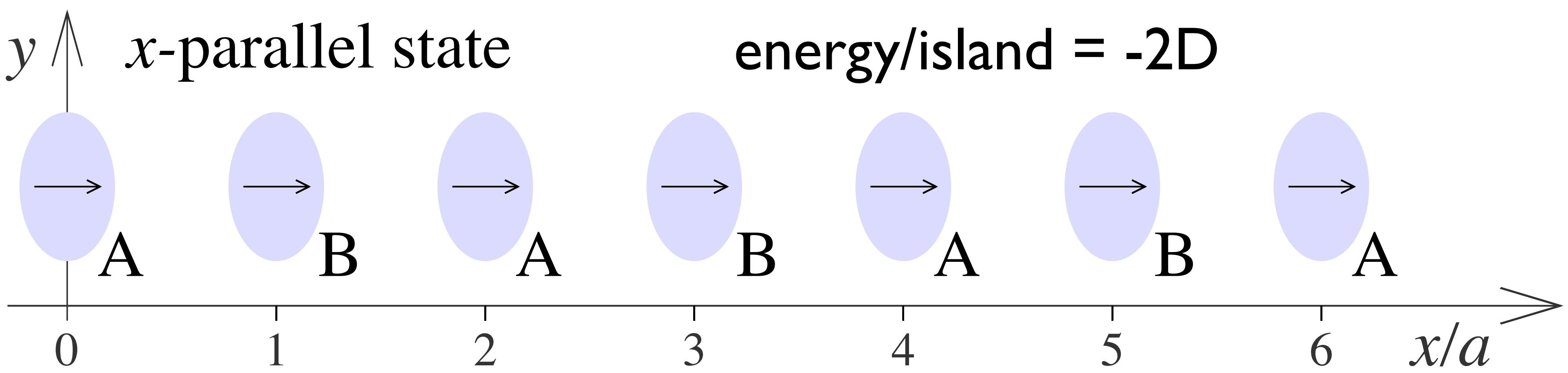


Is this always the lowest energy state?

Note that the system is frustrated! The anisotropy energies are low, but not even the nearest-neighbor dipolar interactions are minimized.

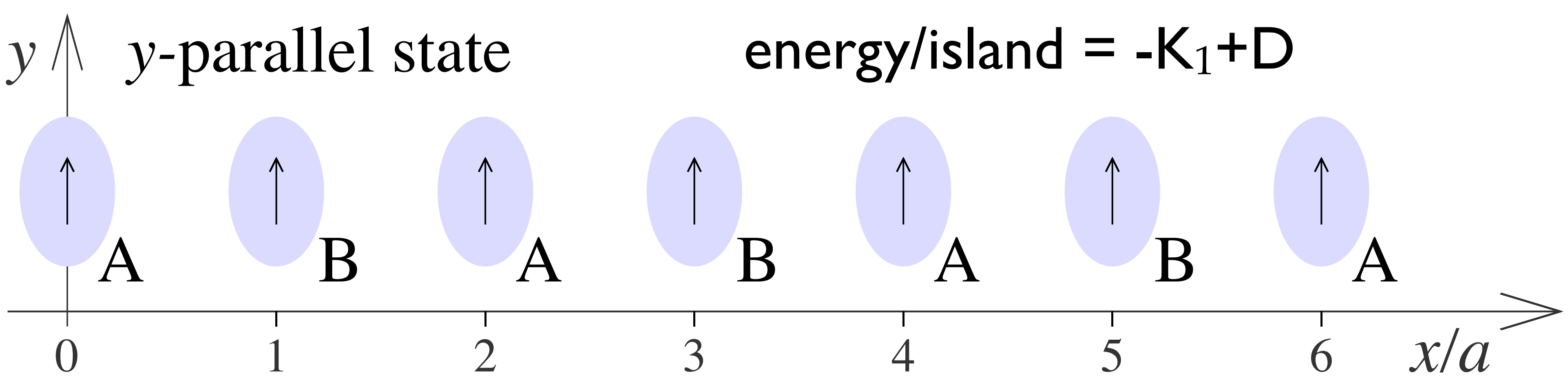
Consider how changing D and K_1 affects the states.

A state that does minimize all the dipolar interactions:

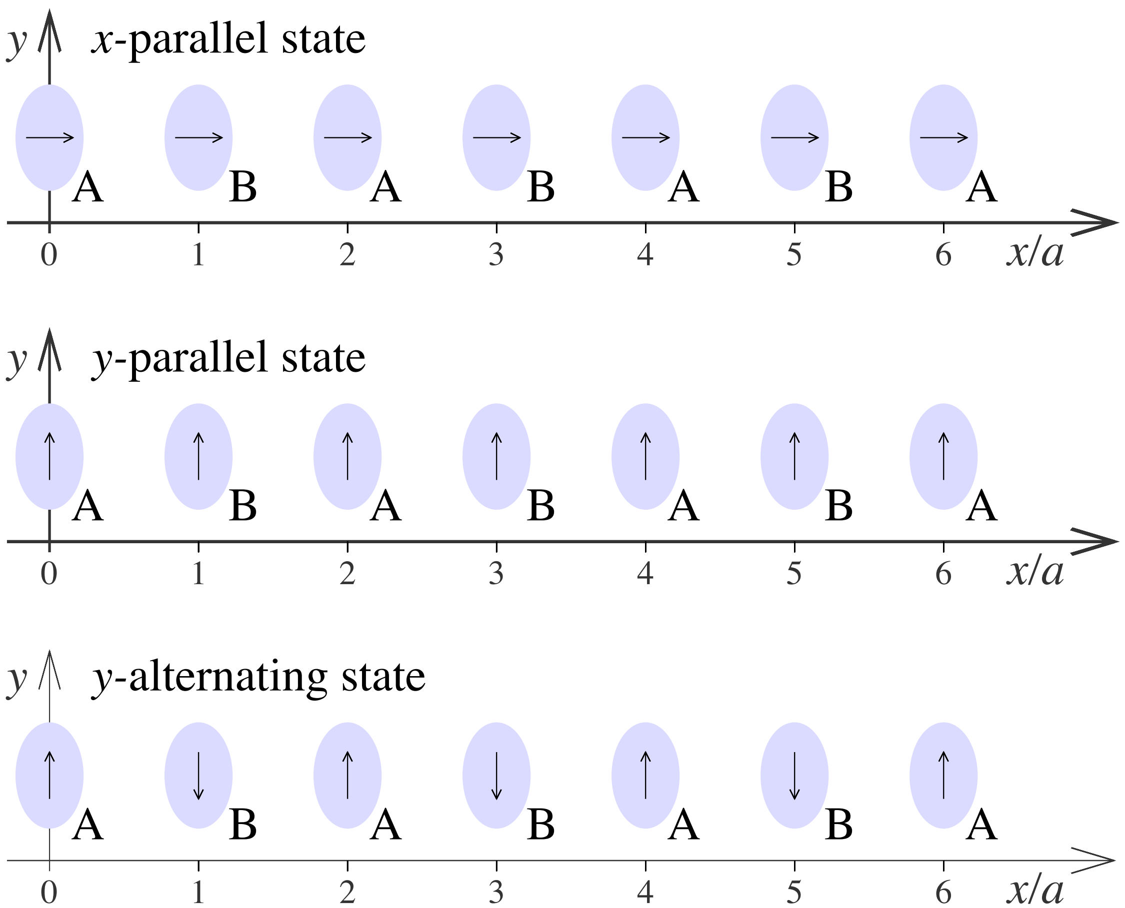


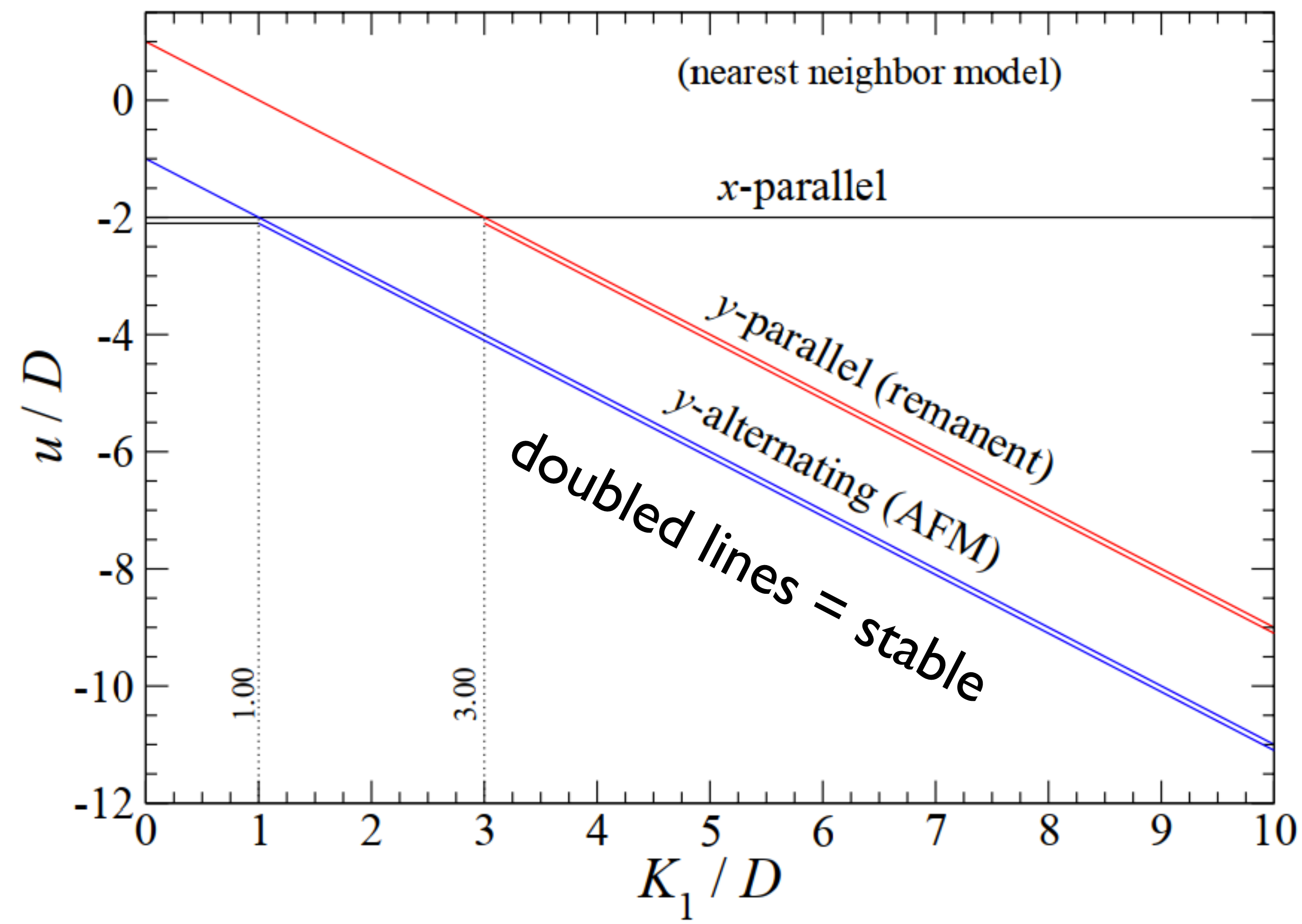
But now the K_1 anisotropy energy is high. ==> Frustration.

A 3rd state, that only minimizes the anisotropy energy:



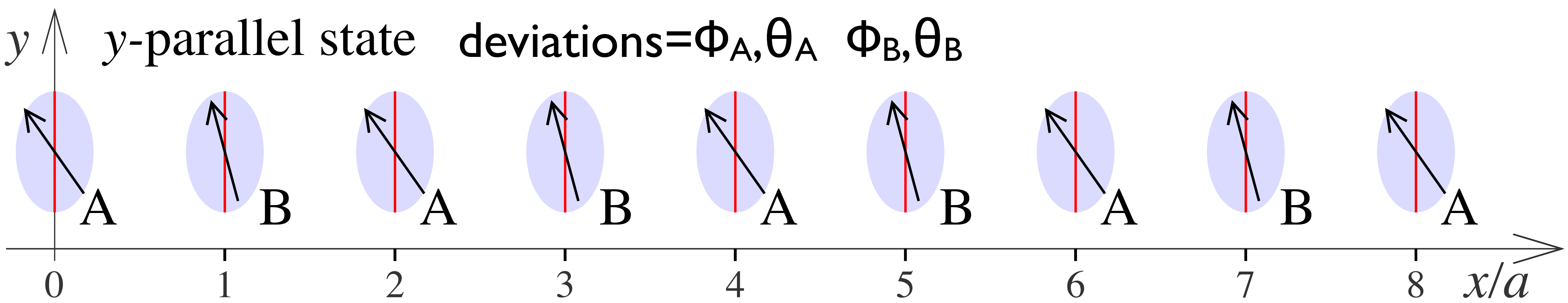
Comparing these uniform states: u = energy/island.





$$
\begin{aligned}
x\text{-parallel:}\quad & u(0,0) = -2D, && K_1 < D, \\
y\text{-parallel:}\quad & u\left(\tfrac{\pi}{2},\tfrac{\pi}{2}\right) = -K_1 + D, && K_1 > 3D, \\
y\text{-alternating:}\quad & u\left(\tfrac{\pi}{2},-\tfrac{\pi}{2}\right) = -K_1 - D, && K_1 > D.
\end{aligned}
$$

Stability verification.
Imagine small angular deviations in the spins, away from the current state.
How does system energy change for *small* sublattice deviations?



Energy changes are decoupled to quadratic order:

$$H_\phi = \psi_\phi^\dagger M_\phi \psi_\phi = \begin{pmatrix} \phi_A & \phi_B \end{pmatrix} \begin{pmatrix} -D+K_1 & -2D \\ -2D & -D+K_1 \end{pmatrix} \begin{pmatrix} \phi_A \\ \phi_B \end{pmatrix}$$

$$H_\theta = \psi_\theta^\dagger M_\theta \psi_\theta = \begin{pmatrix} \theta_A & \theta_B \end{pmatrix} \begin{pmatrix} -D+K_1+K_3 & D \\ D & -D+K_1+K_3 \end{pmatrix} \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix}$$

$$H_\phi = \psi_\phi^\dagger M_\phi \psi_\phi = \begin{pmatrix} \phi_A & \phi_B \end{pmatrix} \begin{pmatrix} -D+K_1 & -2D \\ -2D & -D+K_1 \end{pmatrix} \begin{pmatrix} \phi_A \\ \phi_B \end{pmatrix}$$

Need the matrix eigenvalues > 0 for stability.

$$\sigma_\phi^+ = -3D + K_1, \quad \sigma_\phi^- = D + K_1$$

Requires $K_1 > 3D$

$$H_\theta = \psi_\theta^\dagger M_\theta \psi_\theta = \begin{pmatrix} \theta_A & \theta_B \end{pmatrix} \begin{pmatrix} -D+K_1+K_3 & D \\ D & -D+K_1+K_3 \end{pmatrix} \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix}$$

$$\sigma_\theta^+ = K_1 + K_3, \quad \sigma_\theta^- = -2D + K_1 + K_3$$

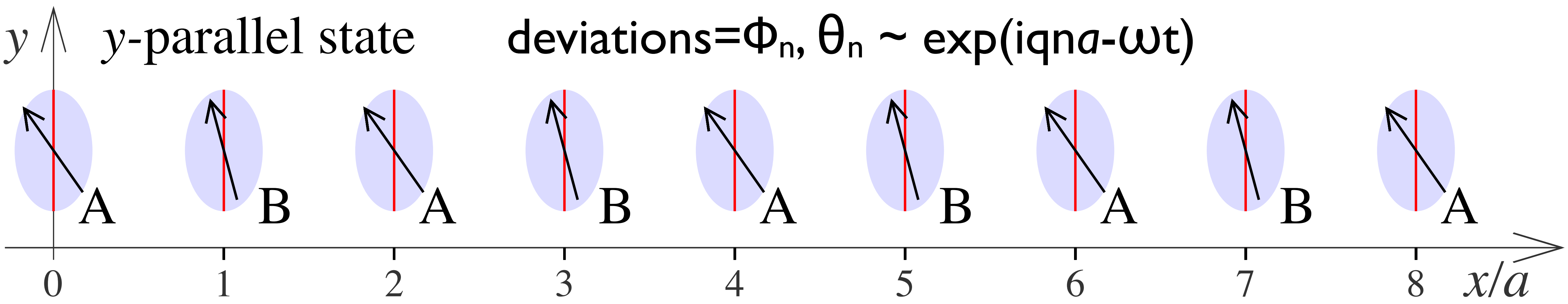
Requires $K_1 + K_3 > 2D$

Conclusion: y-parallel state requires $K_1 > 3D$ for stability.

x-parallel state requires $K_1 < D$ for stability.

y-alternating state requires $K_1 > D$ for stability.

How does the system energy change for a *traveling wave* deviation?



Time dynamics:

$$\frac{1}{\gamma}\dot{\vec{\mu}}_n = \vec{\mu}_n \times \left(-\frac{\partial H}{\partial \vec{\mu}_n}\right) \quad \text{or} \quad \dot{\mathbf{S}} = \mathbf{S} \times \mathbf{F}.$$

Becomes Hamiltonian dynamics, for *small* deviations:

$$\dot{\phi}_n = +\frac{\partial H}{\partial \theta_n} \qquad \dot{\theta}_n = -\frac{\partial H}{\partial \phi_n}$$

Φ_n=coordinate, θ_n=momentum

Deviations: $\psi_\phi^\dagger = (\phi_1, \phi_2, \phi_3, ...\phi_N)$, $\psi_\theta^\dagger = (\theta_1, \theta_2, \theta_3, ...\theta_N)$

The energy: $H = \psi_\phi^\dagger M_\phi \psi_\phi + \psi_\theta^\dagger M_\theta \psi_\theta$

Nearest neighbors only:

$$H = \sum_n \left[M_{\phi,0}\phi_n^2 + 2M_{\phi,1}\phi_n\phi_{n+1} + M_{\theta,0}\theta_n^2 + 2M_{\theta,1}\theta_n\theta_{n+1}\right]$$

$$\dot{\phi}_n = +\frac{\partial H}{\partial \theta_n} = +2M_{\theta,0}\theta_n + 2M_{\theta,1}(\theta_{n-1} + \theta_{n+1})$$

$$\dot{\theta}_n = -\frac{\partial H}{\partial \phi_n} = -2M_{\phi,0}\phi_n - 2M_{\phi,1}(\phi_{n-1} + \phi_{n+1})$$

Solved by 1D traveling waves,

$\Phi_n = \Phi \exp[i(qna-\omega t)]$, $\theta_n = \theta \exp[i(qna-\omega t)]$.

amplitudes wave vector frequency

dispersion relations:

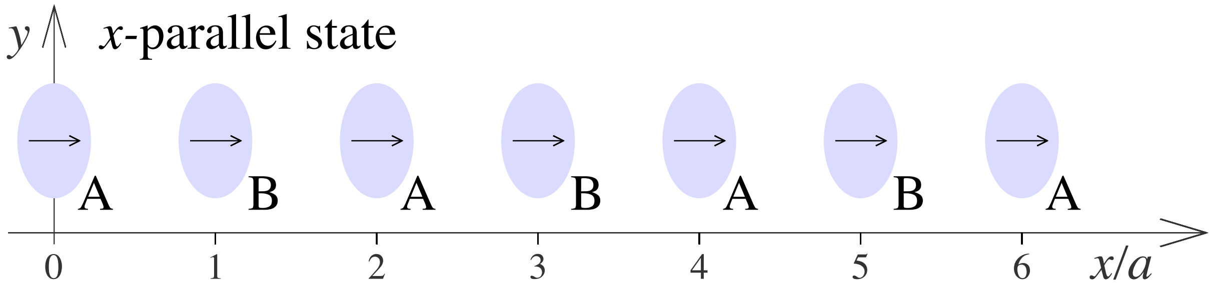
$$\omega^{(m)} = 2\frac{\gamma}{\mu}\sqrt{\lambda_\phi^{(m)}\lambda_\theta^{(m)}}$$

m-th eigenvalues of M_ϕ and M_θ

eigenvalues determined by wave vector:

$$\lambda_\phi(q) = M_{\phi,0} + 2M_{\phi,1}\cos\, qa,$$
$$\lambda_\theta(q) = M_{\theta,0} + 2M_{\theta,1}\cos\, qa.$$

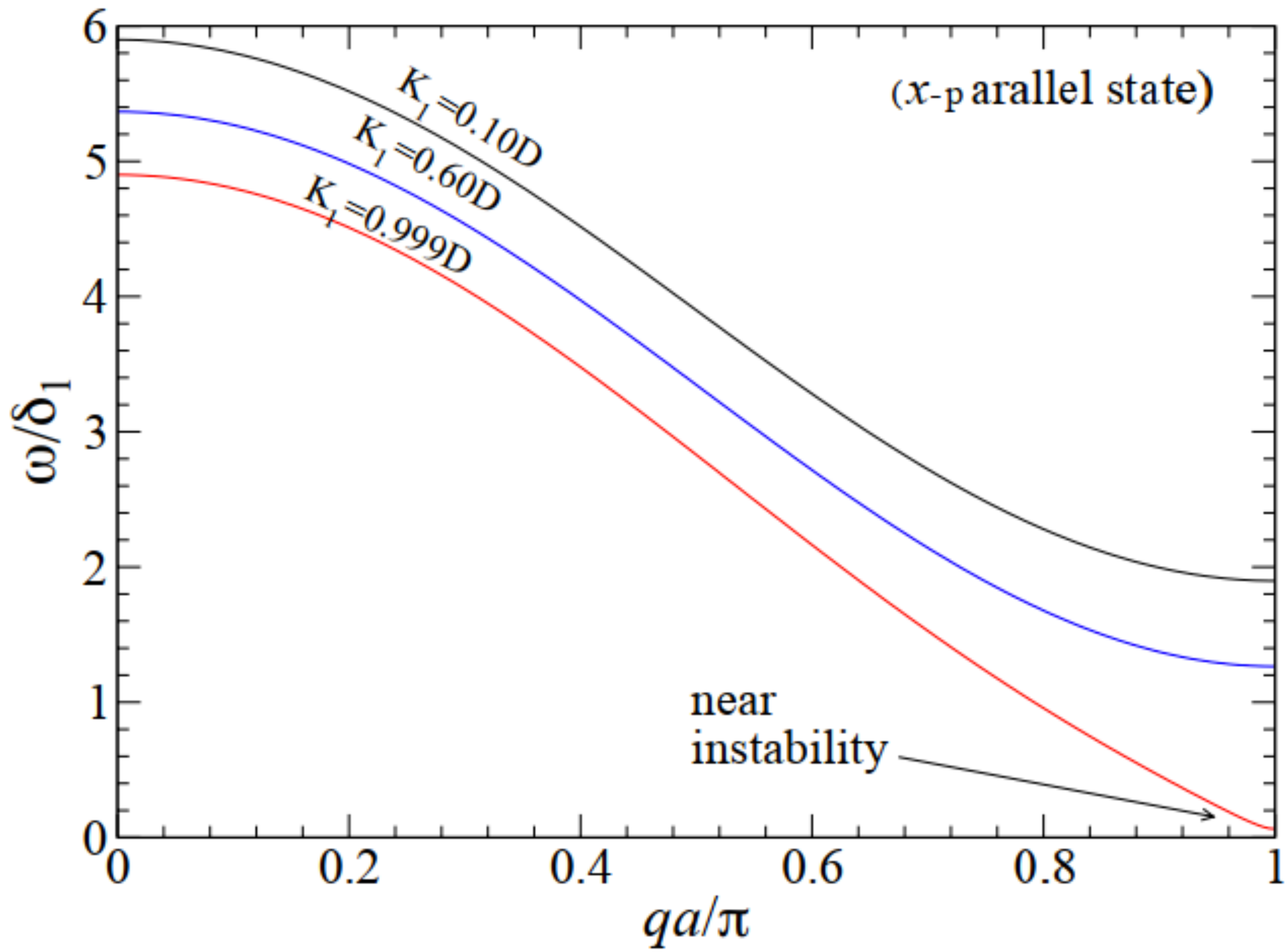
Constants determined by the state,
are functions of D, K_1, K_3.



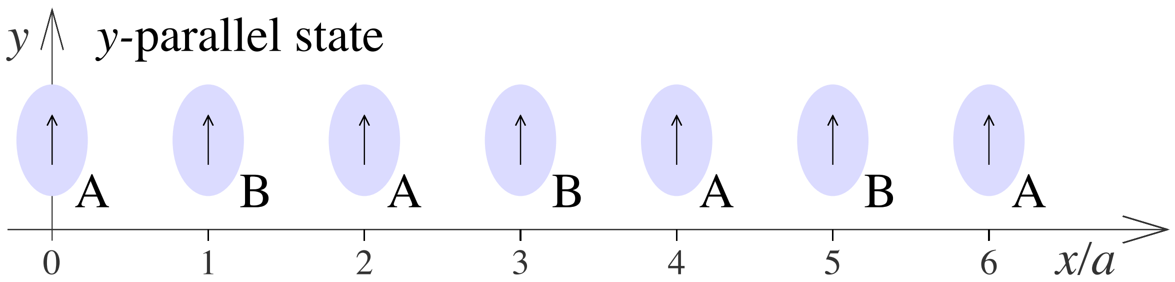
$$\frac{\omega}{4\delta_1} = \sqrt{\left(1+\tfrac{1}{2}\cos qa - \tfrac{K_1}{2D}\right)\left(1+\tfrac{1}{2}\cos qa + \tfrac{K_3}{2D}\right)}.$$

$$\delta_1 \equiv \frac{\gamma D}{\mu}$$

(nearest neighbor dipole interactions)



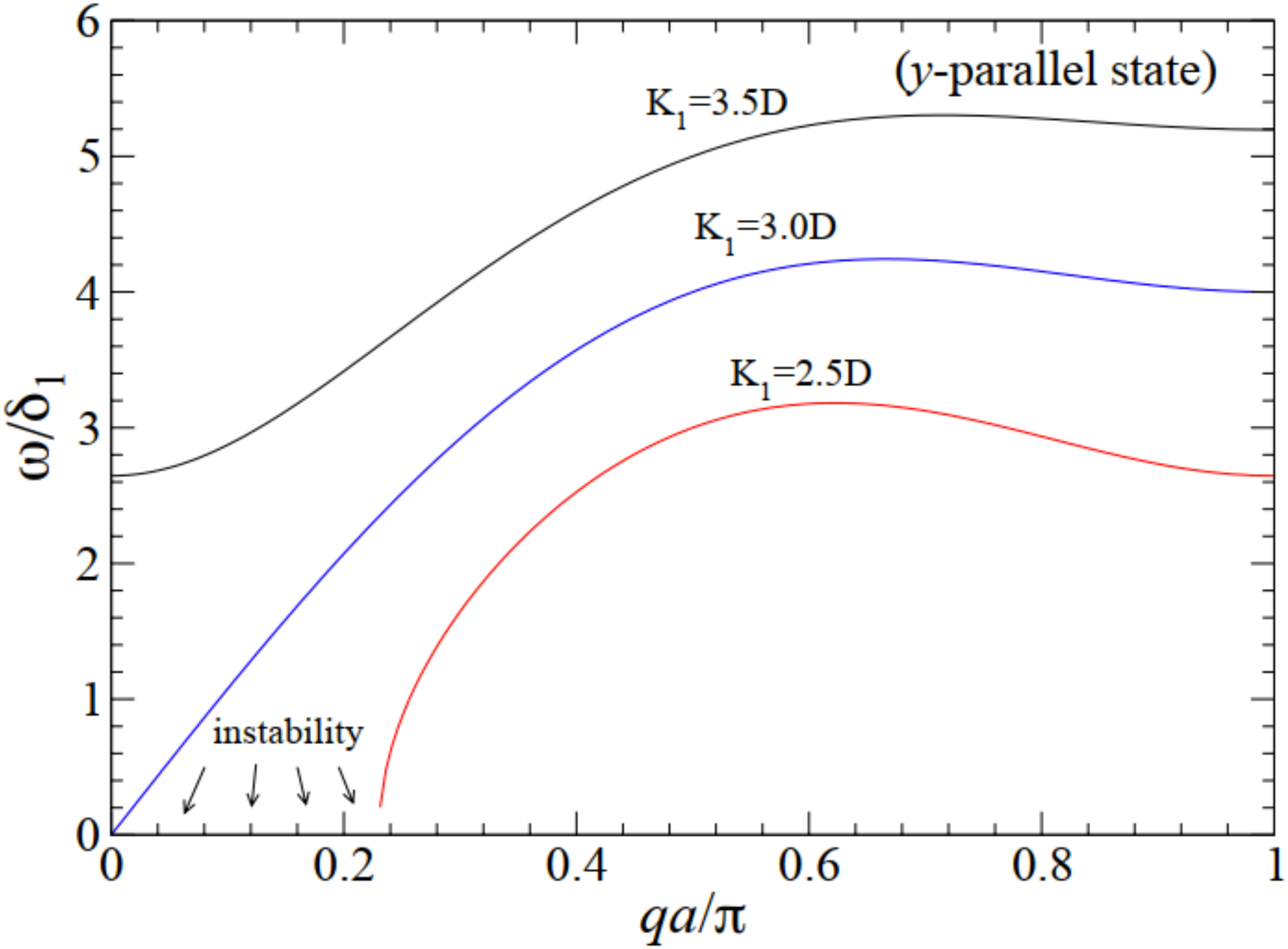
oscillations on top of an x-parallel state.



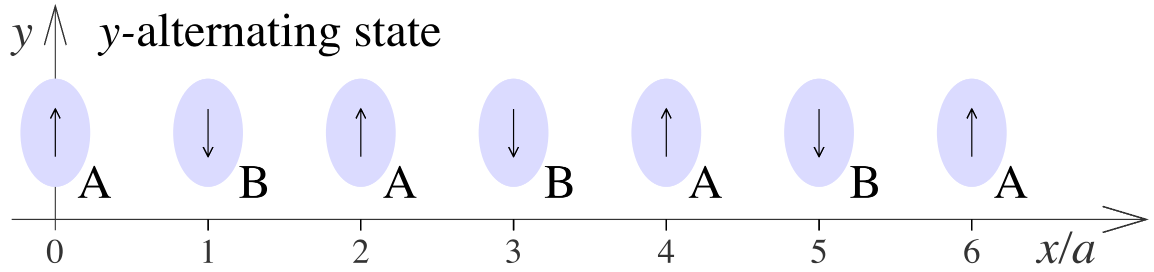
$$\frac{\omega}{2\delta_1} = \sqrt{\left(1 + 2\cos qa - \frac{K_1}{D}\right)\left(1 - \cos qa - \frac{K_{13}}{D}\right)}.$$

$$\delta_1 \equiv \frac{\gamma D}{\mu}$$

(nearest neighbor dipole interactions)



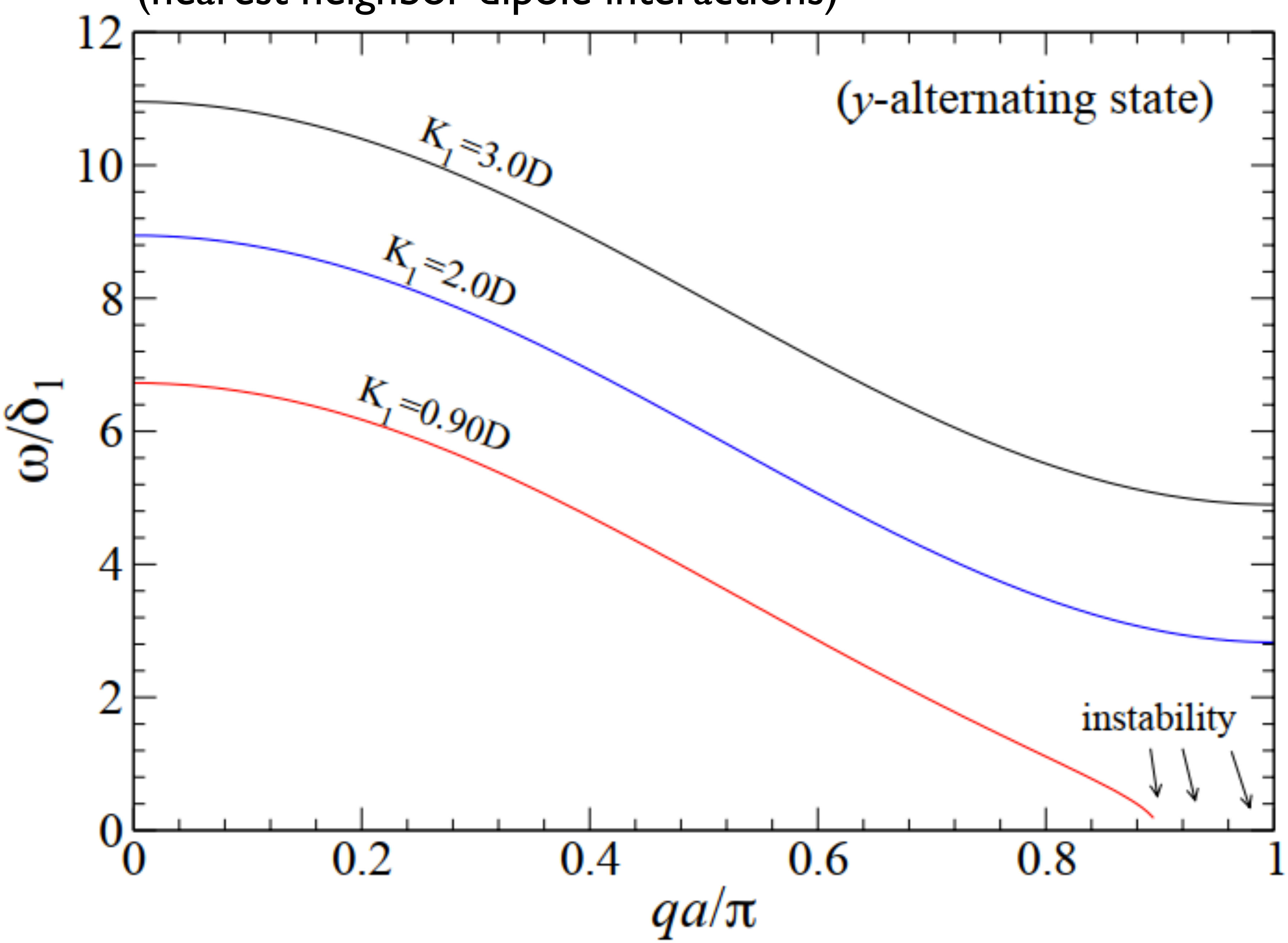
oscillations on top of a y-parallel state.



$$\frac{\omega_g}{2\delta_1} = \sqrt{\left(1 + 2\cos qa + \frac{K_1}{D}\right)\left(1 + \cos qa + \frac{K_{13}}{D}\right)}.$$

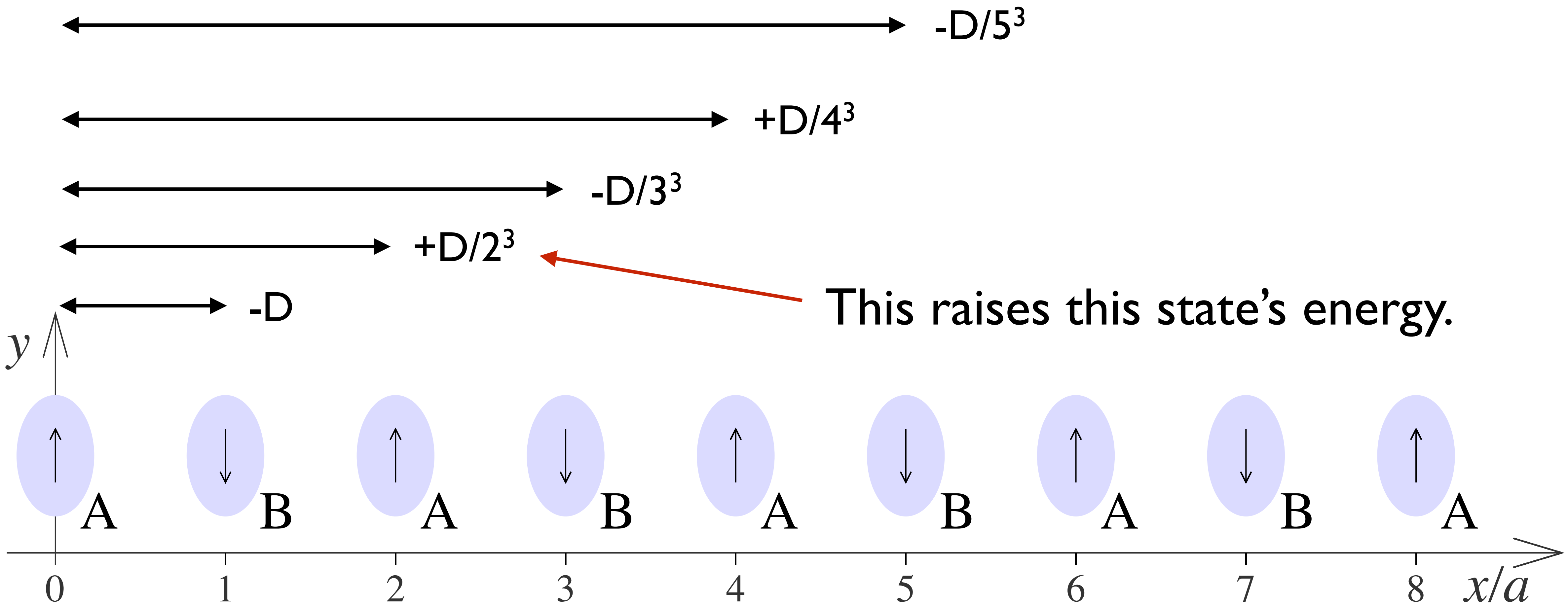
$$\delta_1 \equiv \frac{\gamma D}{\mu}$$

(nearest neighbor dipole interactions)



oscillations on top of a y-alternating state.

But dipole interactions are long range. $H_{dd} \sim 1/r^3$

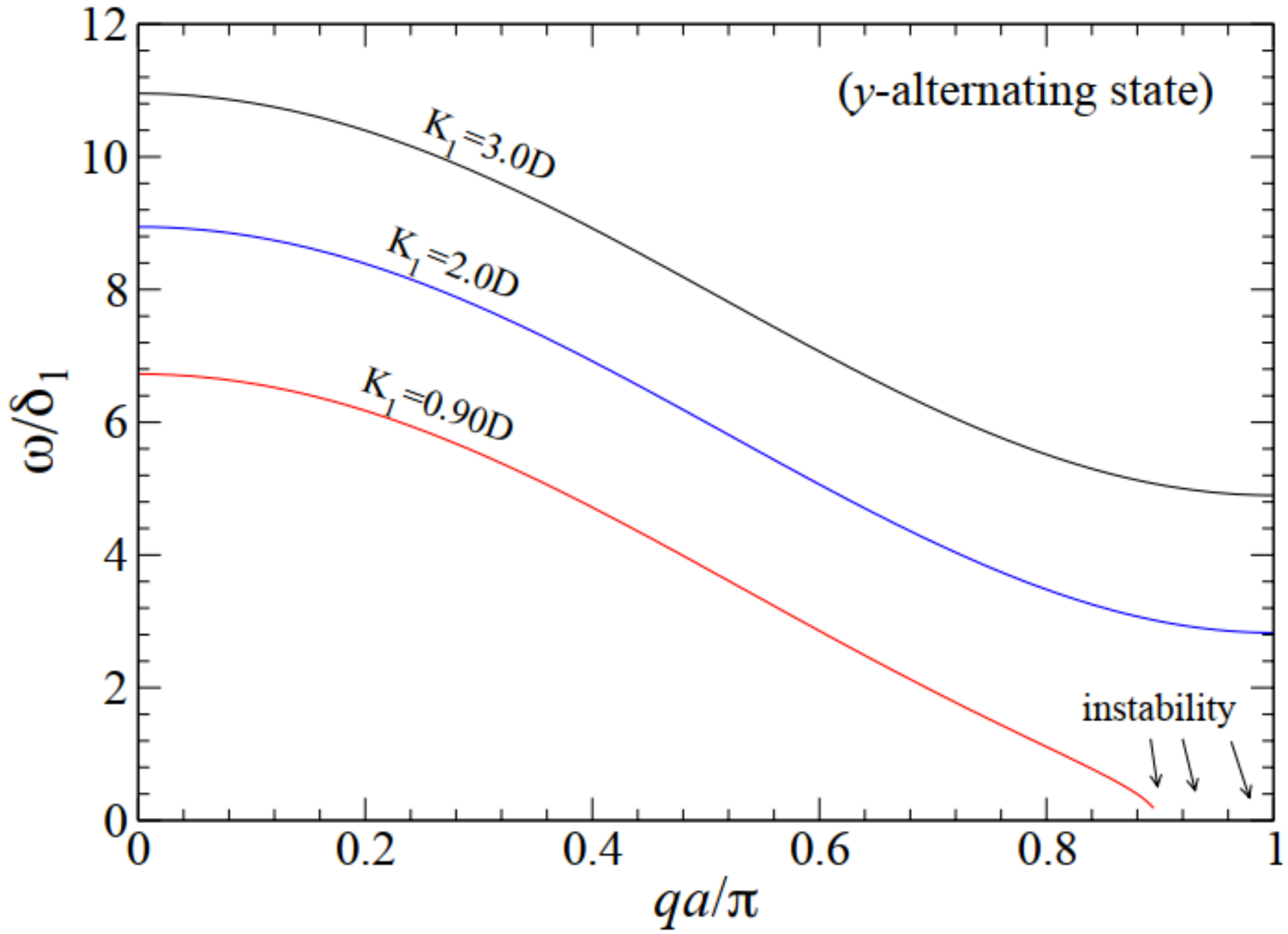


The sums can be done to get energies with all long-range dipole interactions.

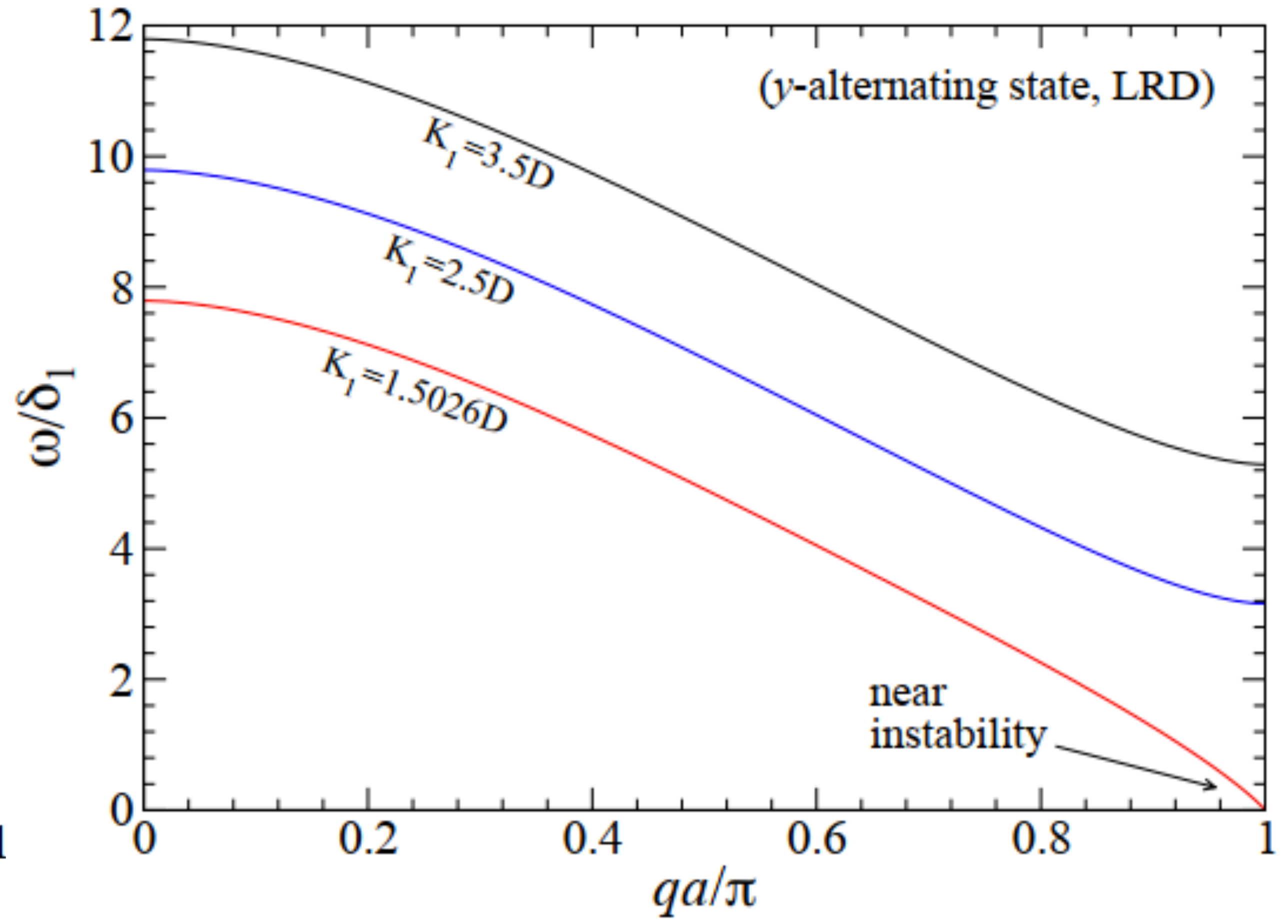
The dynamics can also be analyzed with all long-range dipole interactions!

Dynamic oscillations with long-range dipole interactions.

(nearest-neighbor dipole interactions)

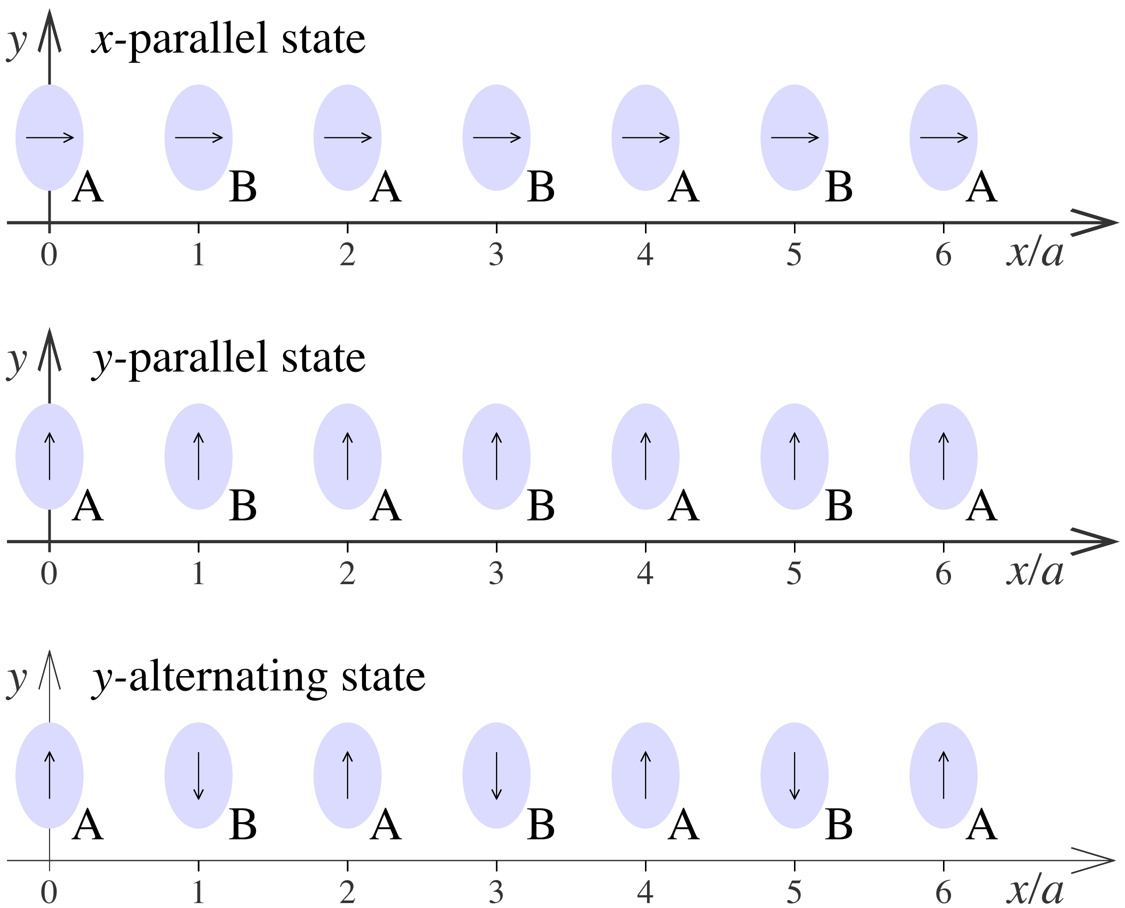


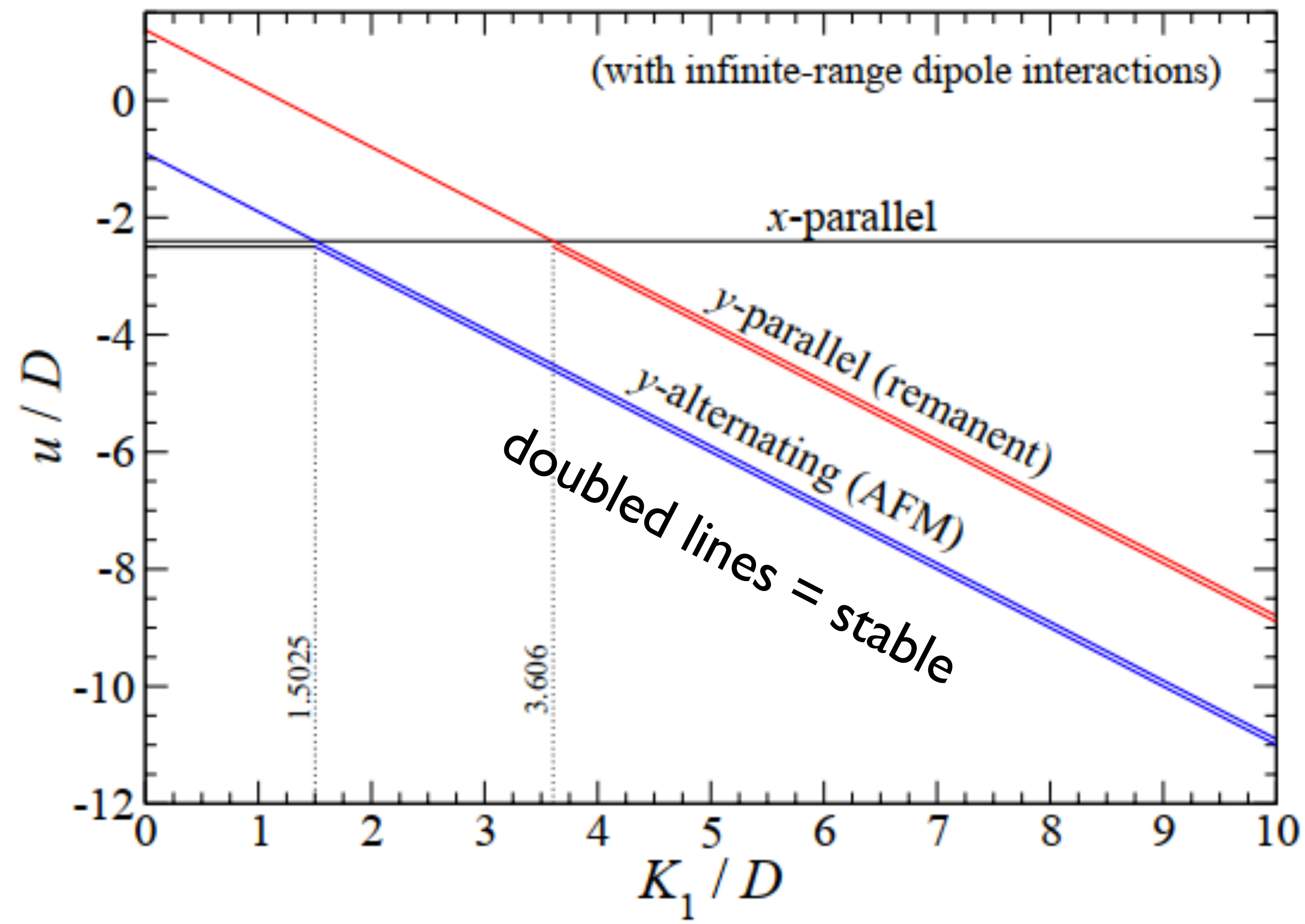
(infinite-range dipole interactions)



Long-range dipole interactions necessitate larger K_1 for its stability.

With long-range dipole interactions: u = energy/island.



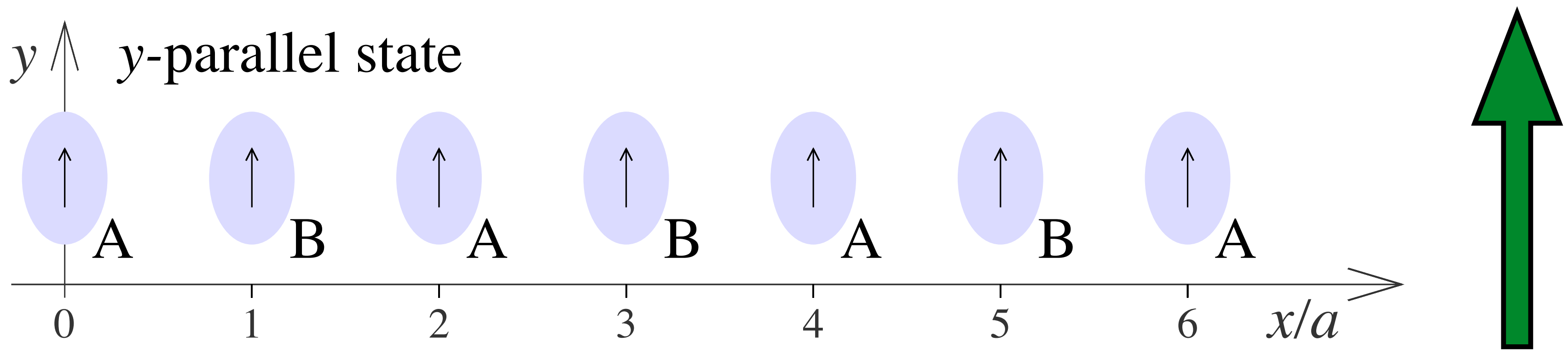


x-parallel: $u = \sum_{k=1}^{\infty} \frac{-2D}{k^3} = -2D\zeta(3) \approx -2.404\, D.$

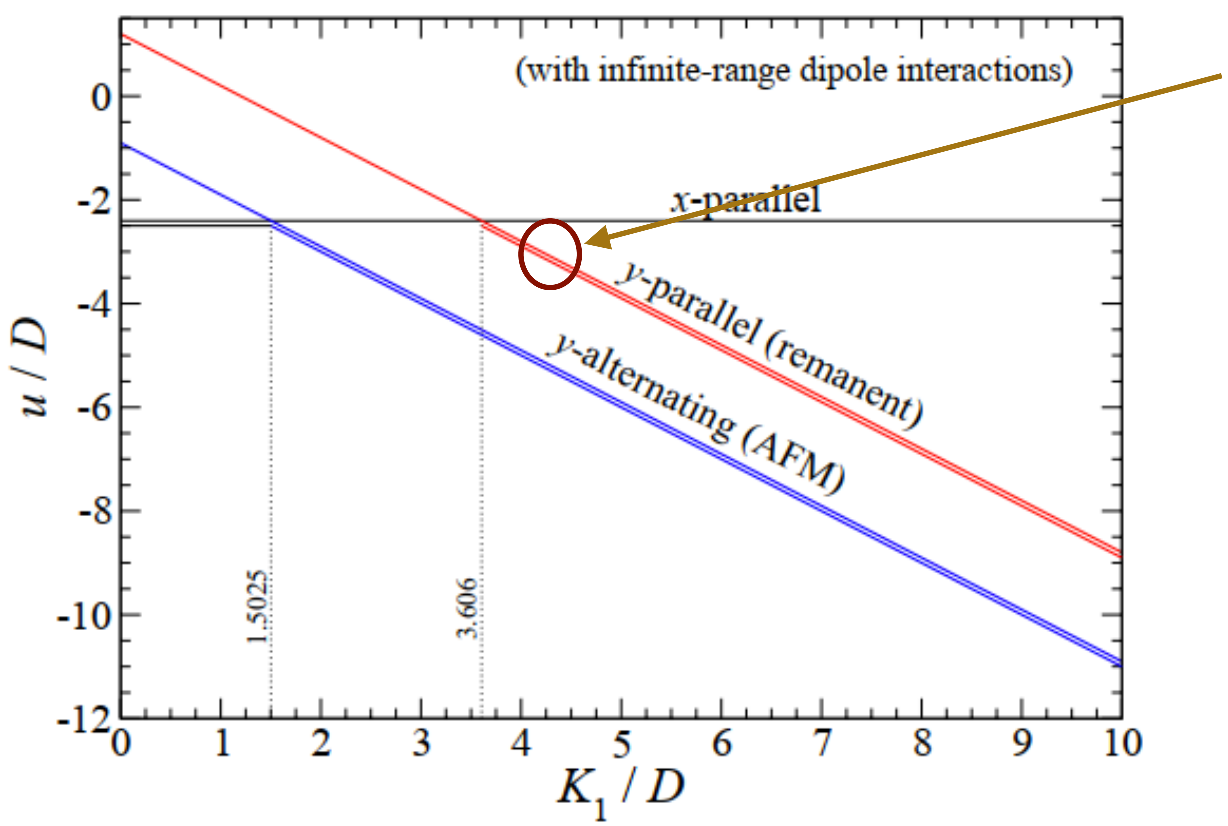
y-parallel: $u = -K_1 + \sum_{k=1}^{\infty} \frac{D}{k^3} \approx -K_1 + 1.202\, D.$

y-alternating: $u = -K_1 + \sum_{k=1}^{\infty} \frac{(-1)^k D}{k^3} \approx -K_1 - 0.9015\, D.$

Application of metastable y-parallel state?



Apply and then turn off B.



Set system here.

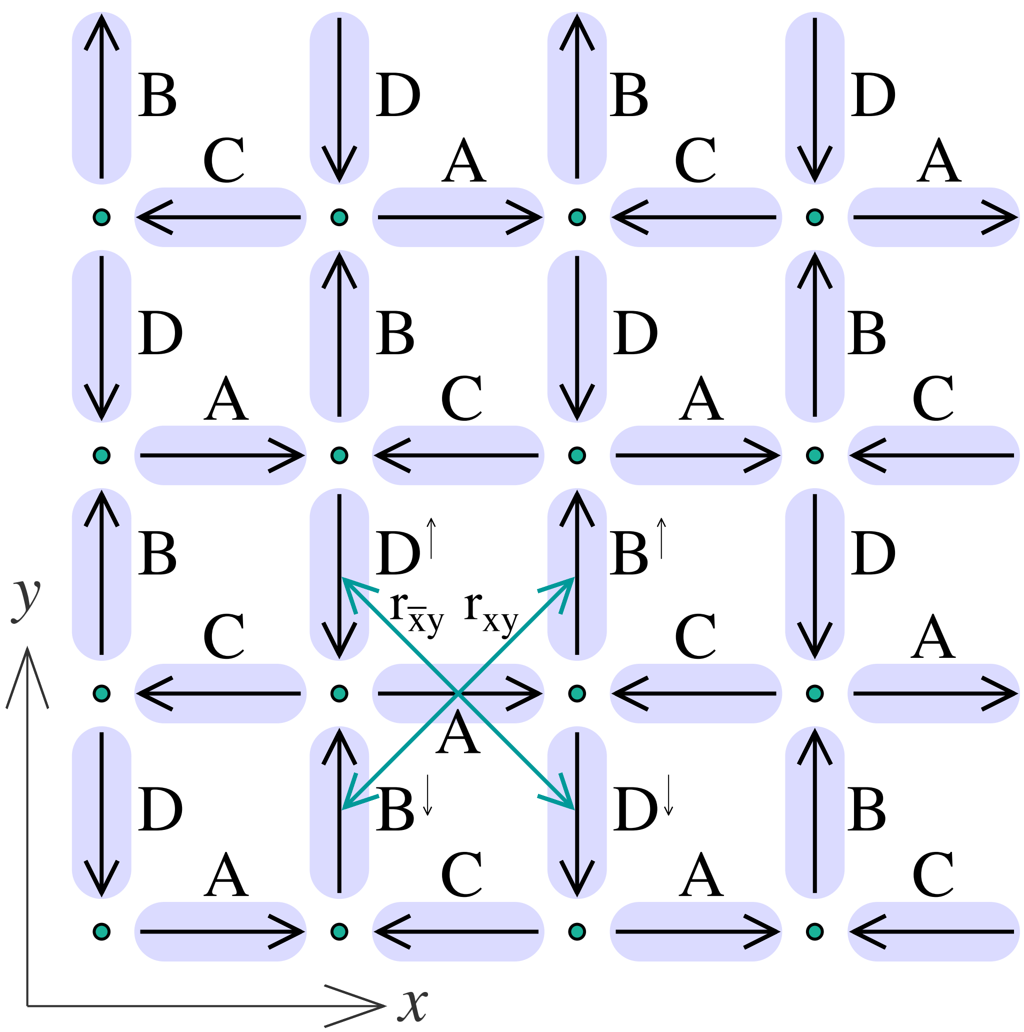
It is marginally stable.

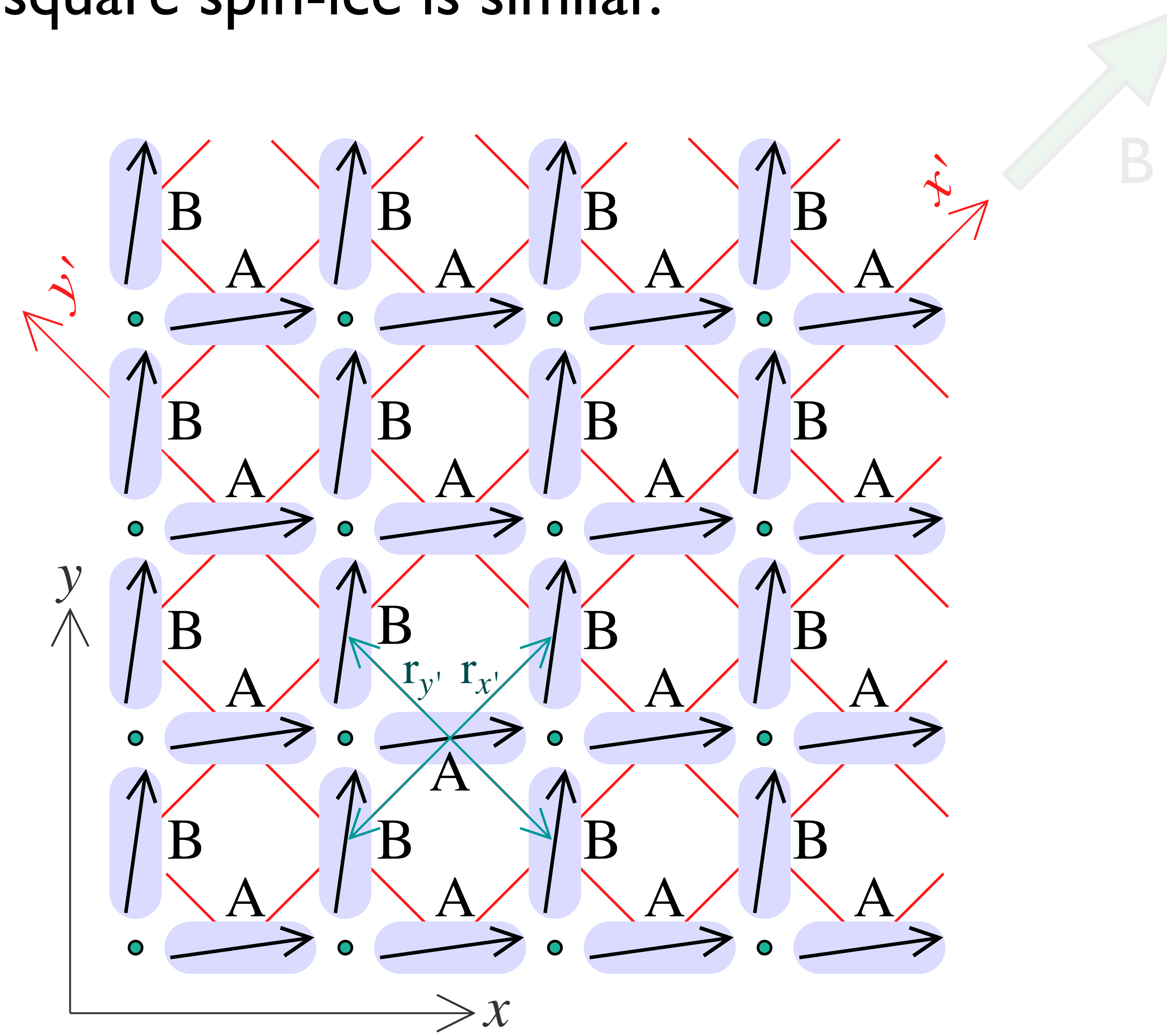
An increase in D could destabilize it and drop system to the y-alternating state.

Some other small perturbation could have the same result. ==> event detector.

Use a B-field to go back to y-parallel.

Metastability of a remanent state of square spin-ice is similar.





A square ice ground state.
There are four sublattices.
Difficult to achieve, due to frustration.

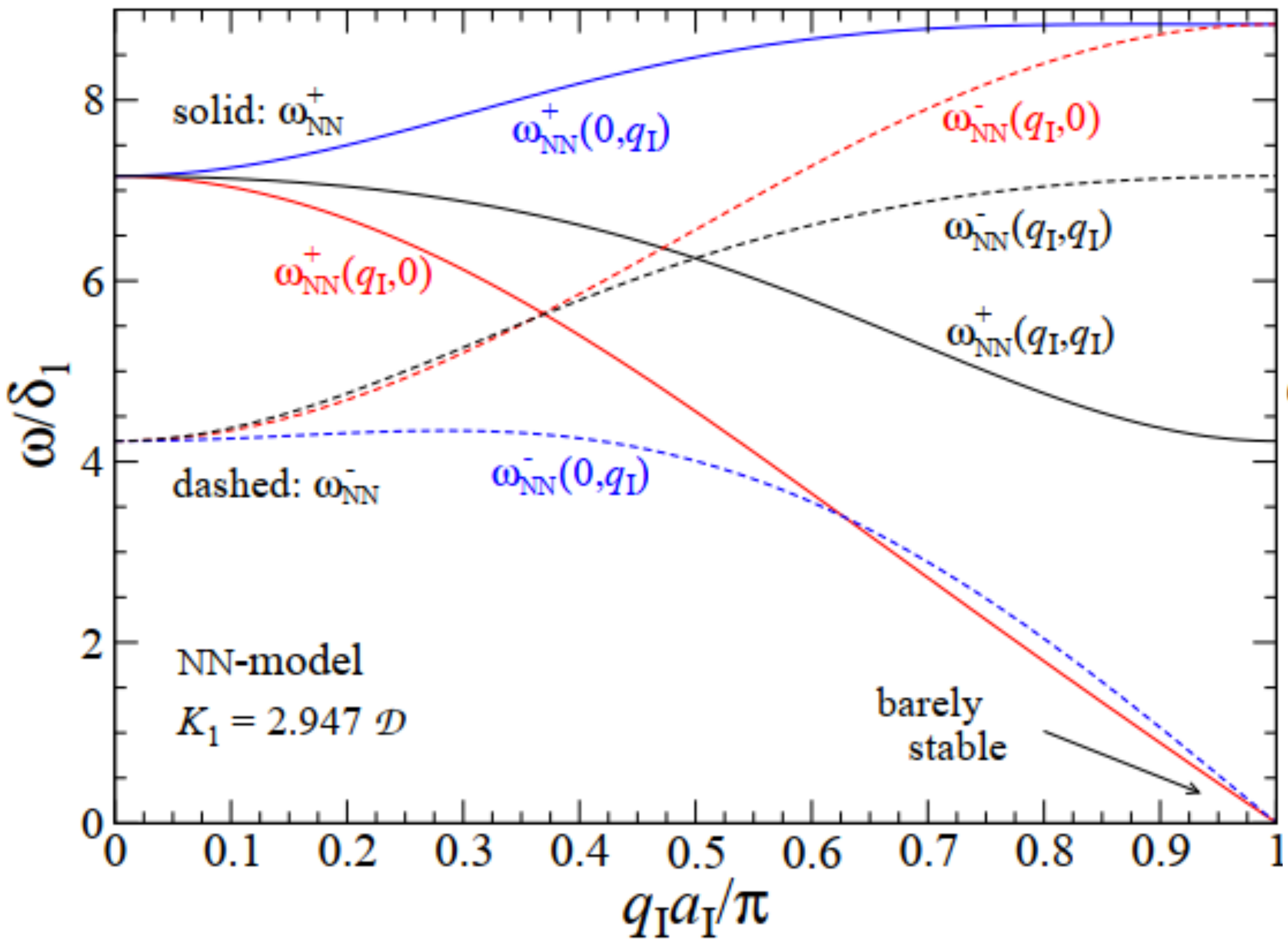
nn energy/island = $\varepsilon_{GS} = -3D$

A square ice remanent state.
There are two sublattices.
Easy to achieve by applied B along x'.

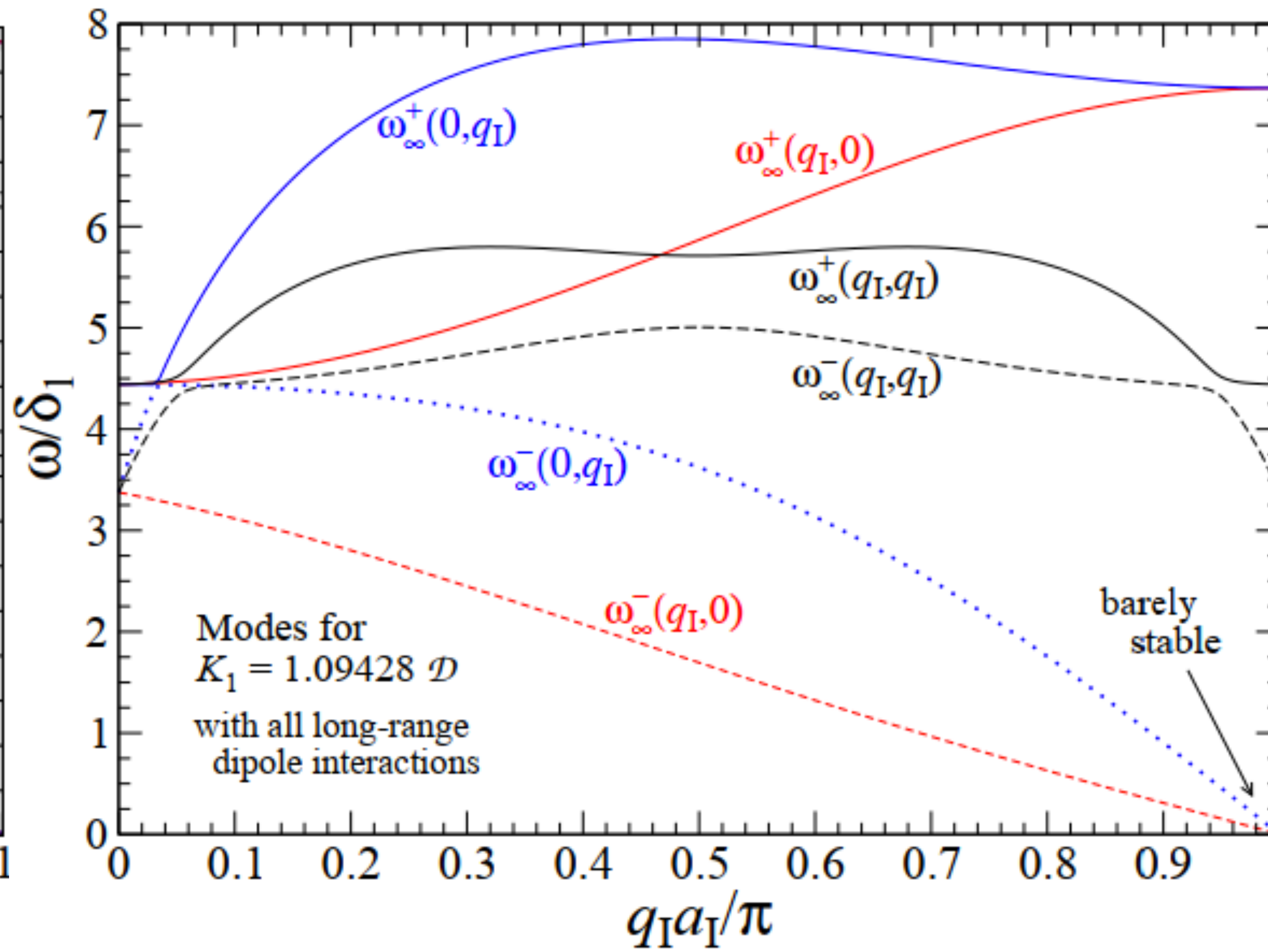
nn energy/island = $\varepsilon_{RS} = -D^2/K_1 > \varepsilon_{GS}$

Oscillations of a remanent state of square spin-ice.

(nearest-neighbor dipole interactions)



(infinite-range dipole interactions)



Long-range dipole interactions help to stabilize the remanent state. Less anisotropy is needed.

SUMMARY:

Lattices of magnetic islands offer a wide range of possible geometries, including chains and spin-ices.

Competing interactions, determined by geometry, imply frustration: not all interaction energies can be minimized.

Models for these systems are used to find some of the uniform states.

Stability is related to eigenvalues for the deviations from a state.

Some small-amplitude oscillation frequency goes to zero at a stability limit.

See publications 91 (1D model), 92 (2D remanent state) at:
https://www.phys.ksu.edu/personal/wysin/publist.htm