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Soliton Dynamics on a Ferromagnetic Chain

G. Wysin[+] and A.R. Bishop[*]
Theoretical Division and Center for Nonlinear Studies
Los Alamos National Laboratory
Los Alamos, NM 87545 U.S.A.

and

P. Kumar[*,++]
Physics Department
University of Florida
Gainesville, FL 32611 U.S.A.

[+]Present address: Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, N.Y. 14853, U.S.A.
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Abstract

We present the results of a numerical simulation of spin dynamics on a ferromagnetic chain subject to an easy plane anisotropy and a magnetic field. The results show substantial deviation from the conventional sine-Gordon description. The soliton like solutions have a variety of effects including large off-easy plane deviation, temporal oscillations and shock-wave formation.

Soliton Dynamics on a Ferromagnetic Chain

The model we consider here, consists of a chain of spins, interacting with a nearest neighbor ferromagnetic exchange interaction and subject to an anisotropy field, perpendicular to the chain direction. In addition, the spins also experience an external magnetic field in the easy plane. Such a model is interesting for two reasons. On the one hand, the model describes the thermodynamics of linear chain ferromagnets (e.g. $CsNiF_3$) at temperatures larger than their bulk ordering temperature (approx. 2K). On the other hand, continuum versions of this model admit both linear (magnons) and non-linear solitons and breathers) excitations that can be analyzed and understood relatively easily. The importance of a theoretically tractable model, describing an experimentally accessible system including the nonlinear excitations and their interactions, is clearly central to developing the understanding of nonlinear phenomena. In particular, the nonlinear excitations here are frequently taken to be described by sine-Gordon equation. The theoretical understanding to date, and comparison with experiments have been reviewed by Steiner (1980) and Mikeska (1980).

Recently, it was pointed out by Kumar (1981a, 1981b) and by Magyari and Thomas (1981) that the sine-Gordon description of this model becomes inadequate at modest magnetic fields and soliton velocities. If the magnetic field is small, the anisotropy field limits the spins largely to a plane. The nonzero winding number excitations then are solitons and are described by the sine-Gordon equation. A moving soliton has small excursions away from this plane, the excursion angle is proportional to the soliton velocity. If the external field is comparable to the anisotropy field (or as shown a fraction larger than .4), the spins are unstable towards rather large off-plane excursions. The texture narrows and becomes immobile. As expected, this instability is aided by the soliton motion which causes spins to go off-plane, such that the critical magnetic field for instability rapidly decreases with increasing soliton velocity.

Whereas the instability calculations above are limited to a continuum version of the discrete chain, we have carried out a numerical simulation of soliton dynamics on a discrete chain, without any sine-Gordon assumptions, to understand (a) whether there are any serious discrete lattice effects on the instability and (b) the nature of the nonlinear excitations after the instability has occurred. The Hamiltonian is described by (Steiner 1981)

$$H = -J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + A \sum_i (S_i^z)^2 - \sum_i \mathbf{S}_i \cdot \mathbf{B} \tag{1}$$

where the sums run over the lattice sites separated a distance a apart, along the z-axis. J represents the strength of the exchange interaction and A and B respectively the anisotropy and the external magnetic fields. The spin dynamics is described by the Bloch equation

$$\dot{\mathbf{S}}_i = -\gamma\, \mathbf{S}_i \times \frac{\delta H}{\delta \mathbf{S}_i} \tag{2}$$

The continuum limit of Eq.(2) has been analyzed. If the polar and azimuthal angles of spins are θ and ϕ, Eq.(2) can be expressed in terms of the field equations (the lattice spacing is given by a)

$$\sin\theta\dot{\theta} = \frac{\partial E}{\partial \phi} \; ; \quad -\sin\theta\dot{\phi} = \frac{\partial E}{\partial \phi}$$

and

$$E = \int_{-\infty}^{\infty} \frac{dz}{a} \left[\frac{1}{2} Ja^2(\theta_z^2 + \sin^2\theta\phi_z^2) + A\cos^2\theta - B\sin\theta\cos\phi\right] \tag{3}$$

where the magnetic field B is taken in the x-direction. The principal conclusions (Mikeska 1981) in the sine-Gordon limit (low field, low velocity) can be summarized:

(1) The ϕ profile is given by $\sin\phi/2 = \mathrm{sech}(z-ut)/\zeta_0(1-u^2/c^2)^{1/2}$ where u is the soliton velocity, $\zeta_0^2 = Ja^2/H$ and $c^2 = 2AJa^2$.

(b) The off-plane excursion is determined by $\theta = \pi/2 - \frac{u}{2A}\phi_z$.

A linear stability analysis of a static profile shows instability for $B > B_c(0) = 2/3$ A. However in the range $A/2.4 < H < 2/3$ A, the distorted soliton has lower energy. For a slow moving soliton the excursion angle $\psi = \theta - \pi/2$ is given by $\psi = \frac{u}{2A} \cdot \frac{B_c}{B_c - B}\phi_z$ and is always larger than the sG value. At $B = B_c(0)$; $\psi \propto u^n$ where $n < 1/5$ (Kumar 1981b). The critical field for distortion decreases with the soliton velocity as $B_c(u) = B_c(0) - Au^{2/3}$ (Magyari and Thomas 1981).

Our numerical calculations have been done on a chain of 180 spins with periodic boundary conditions and energy conservation. Depending on the magnetic field, static sG solitons vary in width from 10 to 20 spins. Typically at time t=0, a discretized sG soliton is launced with a specified velocity u_{sG} (thereby defining the total energy) and the subsequent time evolution is described by Eq.(2). If the sG approximation is valid then the soliton would propagate with unchanged velocity. We find, on the contrary, that the soliton motion is different at arbitrarily low non-zero* velocities and the difference increases with increasing magnetic field. In Fig.(1), we show the observed velocity u^+ of the soliton as a function of the initial velocity u_{sG} for different magnetic fields. Even at low fields, the final average velocity u is always less than u_{sG}. The velocity u reaches a critical maximum and then begins to decrease with increasing u_{sG}(i.e. energy). This maximum u, which we denote by u_c, decreases with increasing magnetic field.

The initial slope of this curve $\frac{du}{du_{sG}} \alpha$ (B*-B) where B* = .62 A (c.f. B_c = 2/3 A, Kumar 1981a and Magyari and Thomas 1981). The field dependence of u_c is consistent with the MT result in that $B_c(o) - B_c(u) \sim u^{2/3}$ (Fig. 1b). Fig. 1c shows E(U). We refer to solitons with $E < E(u_c)$ and $E > E(u_c)$ as belonging to the lower and upper "branches" respectively (cf. Magyari and Thomas 1981).

We have found no evidence for a discontinuous change in passing from thelower to upper branches at u_c. However, deviations from sG increase continuously as we increase energy on the E(u) curves and these deviations increase most strongly on the upper branch. Thus, as the energy increases the anisotropy energy becomes an increasing fraction of the total energy and begins to participate increasingly in the dynamics. This is consistent with the distorted soliton picture. Similarly (see Fig. 2) the off-plane excursion angle increases with increasing energy and is increasingly greater than the off-plane angle predicted in the sG approximation. The facts that the off-plane excursion angle and the anisotropy energy contributions increase, all with decreasing velocity and that the soliton width decreases on the upper branch are quite inconsistent with the sG approximation and can only be understood in terms of a distorted texture.

The distorted texture remains well defined for $E > E(u_c)$. In as much as the Poincare cycle period for our system is about 600 (in the units used here) we deal with an essentially infinite system. The initial sG profile decays very rapidly (with 10 time steps in a typical run of one or two thousand) into the stable distortions of the system (subject to the boundary conditions $\phi \rightarrow (0,2\pi)$, $z \rightarrow (-\infty,\infty)$). We expect that any further time dependent phenomena is the natural evolution of the system's textures. It is characteristic of our initialization procedure that internal oscillations are excited in the relaxation to the distorted soliton profiles, which are of increasing amplitude as E increases (i.e. as the initial and final states are increasingly different). These oscillations occur sympathetically in the velocity, shape, off-plane angle, etc., but most periodically in energy components and are responsible for magnon generation from the (non-sG) dynamic solitons. We have found that the amplitude of oscillations can be significantly reduced by the use of closer initial conditions, but the frequency of periodic fluctuations and mean values are essentially unchanged (this supports the intrinsic nature of those features). The frequency decreases as E increases on E(u) curves at fixed B and A (Fig. 2), whereas at fixed E and A it is proportional to B.

The mean maximum off-plane angle becomes large for upper-branch solitons with increasing energy (decreasing velocity). For instance with $\frac{g\mu_B}{JS} = 0.012$ and u = 0 (upper branch), the mean maximum off-plane excursion is ~60°. As the input energy increases further, we find a single kink solution which propagates backwards: E(u) is continuous at u=0 — see Fig. 1c. The kink narrows further as the off-plane angle increases further, and the frequency of oscillations (above) further decreases. This surprising backward motion is limited by a shock-like phenomenon (below when the kink width contracts to a lattice spacing and the out-of-plane excursion approaches 90°. However the maximum positive and negative propagation velocites are unrelated. For instance at B = 0.25A (~7 kG in $CsNiF_3$), the forward motion is limited by U_{max} = 0.04 C, whereas the backward motion is possible to -U ≃ 0.8 C. Beyond this speed the texture develops a "shock front". The ϕ profile contracts such that the $(0,2\pi)$ change occurs over a lattice spacing. A nonlinear pulse is emitted which grows into a 2π kink and continues moving backwards with the same speed. After a short time the new kink narrows and the whole process repeats. Backward moving solitons also characterize final states for $B > B_c(0)$ (Fig. 1c), for which there are no lower branches. Backward propagation might be understood as a recoil to the rapid emission of magnons. The "shock" features are necessarily beyond a continuum description. Clearly a symmetry of positive and negative energy velocities is restored by starting with negative velocity initial conditions, but the spectrum of propagating kinks is far richer than the sG approximation can predict.

In summary we have studied numerically possible single kink excitations in the easy-plane ferromagnetic Heisenberg model in an easy plane magnetic field. We have found a multi-branched single soliton excitation structure. Only the lowest branch can be sensibly explained by perturbed sG travelling wave profiles. The more general solutions are far from sG with the possibility of strong oscillatory components and coupling to the translational degrees of freedom — in the presence of strong off-plane spin excursions this is entirely reasonable (c.f. pulse-solitons of the isotropic Heisenberg model): appropriate components of linear and angular momentum are sensible dynamical variables (e.g. Bishop 1980). Our conclusions clearly profoundly affect conventinal sG-based phenomenological descriptions (e.g. for $CsNiF_3$ -- c.f. similar modifications for antiferromagnets (Shiba, et. al. 1981). However, before seriously assessing these it is essential to understand kink-antikink scattering and bound state (breather) dynamics. The situation is strikingly similar to that holding (Mikeska 1981, Sklyanin 1980) with Ising (rather than magnetic field) symmetry-breaking in an easy-plane ferromagnet, but with important differences since that case is completely integrable (in the continuum limit) with the usual consequences for kink and breather dynamics. We will report on our results for these problems, and on the crossover to isotropic Heisoberg,in a succeeding article.

Footnotes

* Static ($u_{sG}=0$) initial sG conditions are preserved (without oscillation) to a high accuracy (i.e. $u=0$), for $B < B_c(0)$. This is expected since the sG description is strictly valid for static (lower branch) solitons in the continuum limit.

\+ We determine soliton "velocity" without sophistication, namely from the π-crossing for the in-plane angle or the maximum in the out-of-plane angle. We do not consider more precise numerical definitions appropriate at this point, but it should be cautioned that asymmetric shape oscillations can lead to apparent translations.

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Figure Captions

Fig. 1.

(i) Numerical procedure: initial conditions of sine Gordon kink with velocity u_{sG} result in kink progating with mean velocity u, which may be negative and oscillates. $A/J = 0.0477$ and $g\mu_B B/2AS =$ (a) 0+ (i.e. sine-Gordon limit), (b) 0.0419, (c) 0.0839, (d) 0.1258, (e) 0.1677, (f) 0.2096, (g) 0.2516, (k) 0.2935.

(ii) Maximum (mean propagation velocity (i.e. bifurcation velocity between "upper" and "lower" branches — see fig. 1(iii)) versus magnetic field.

(iii) Energy versus (mean kink velocity with $A/J = .0477$ and $g\mu_B B/2AS =$ (a) 0+ (i.e. sine-Gordon limit), (b) 0.05, (c) 0.10, (d) 0.15, (e) 0.24. Notice the "upper" and "lower" branches and bifurcation points described in the text. The dashed lines indicate the additional branches found numerically as negative velocity regimes (the figure is symmetric in $u \rightarrow -u$). These terminate with a "shock" phenomenon when the out-of-plane deviation approaches $\pi/2$ (see also Fig. 2(i)). For $B \tilde{>} B_c$ only this branch survives.

Fig. 2.

Properties as functions of progating kink energy for $A/J = 0.0477$ and $g\mu_B B/2AS =$ (a) 0.0419, (b) 0.1258, (c) 0.2096, (d) 0.2935, (e) 0.3354. Case (a) lies entirely on the lower branch, whereas cases (b) - (e) include points on both upper and lower branches.

(i) The maximum (mean) out-of-plane angles θ_m in radiations. Note how rapidly the "instability" limit ($\pi/2$) is reached for $B \sim B_c$.

(ii) The same data as (i) normalized to the predictions for the sine-Gordon limit $\Delta = \theta_m / \theta_m$(sine-Gordon); $[\theta_m(\text{sine-Gordon}) = 2u(1-u^2/c_o^2)^{-1/2}(g\mu_B B/2\lambda S)^{1/2}]$. Note that this always <u>under</u>estimates the out-of-plane deviation.

(iii) The oscillation frequency for energy components. (Time measured in units $(JS)^{-1}$). The <u>amplitude</u> of these oscillations depends on initial conditions but the frequency does not. Additional frequency components appear for kinks near to instability.

FIG. 1 (i)

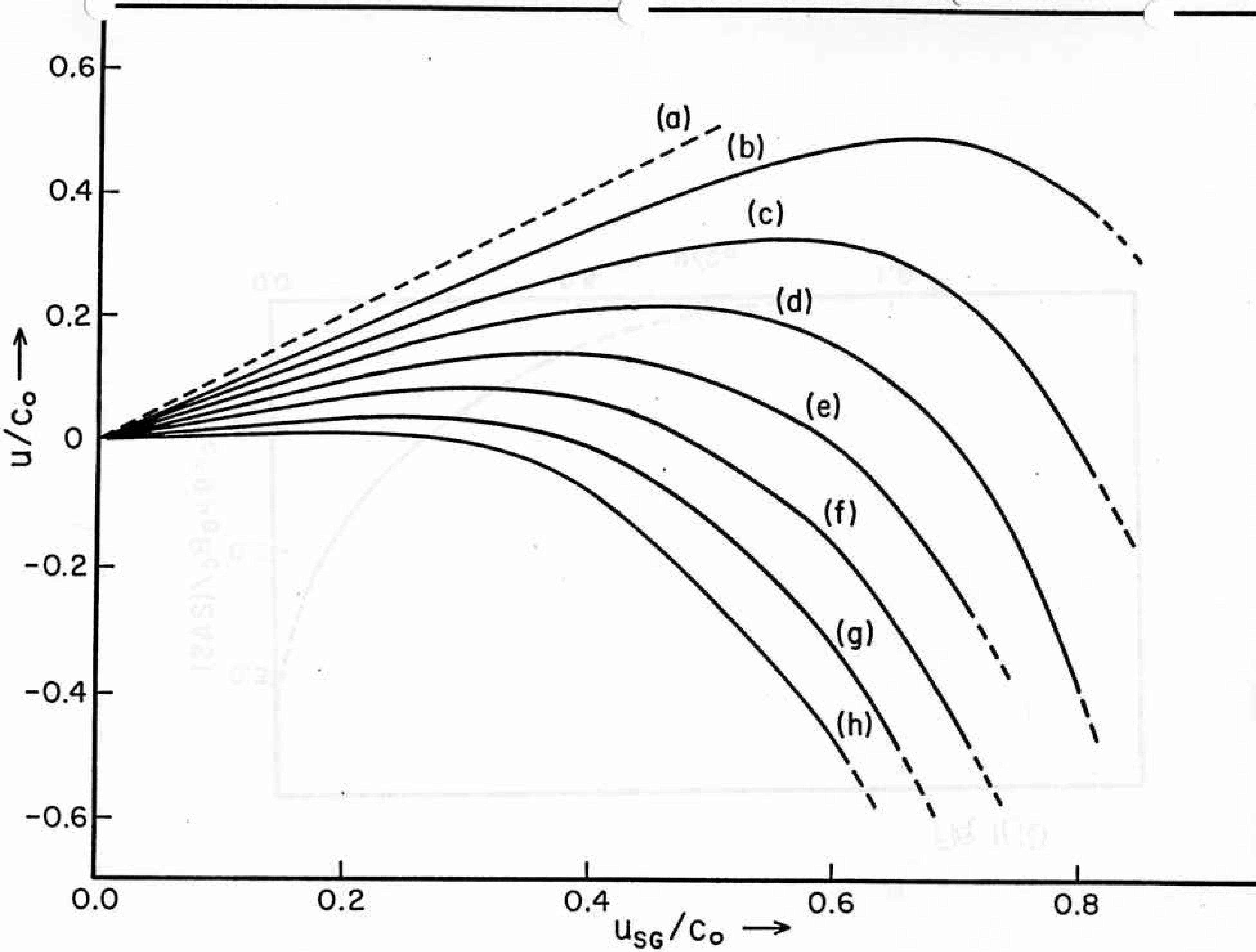


FIG. 1(ii)

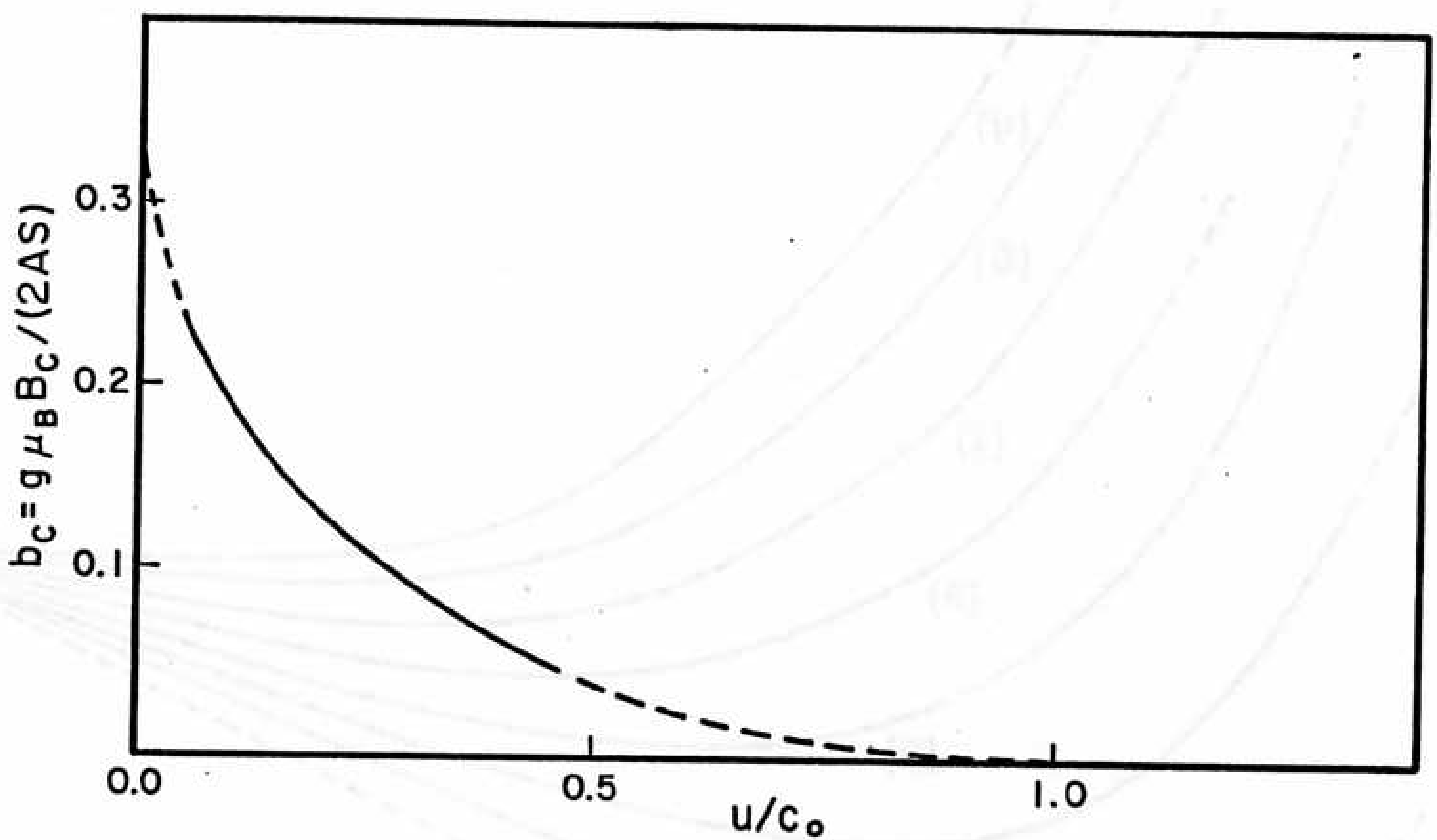
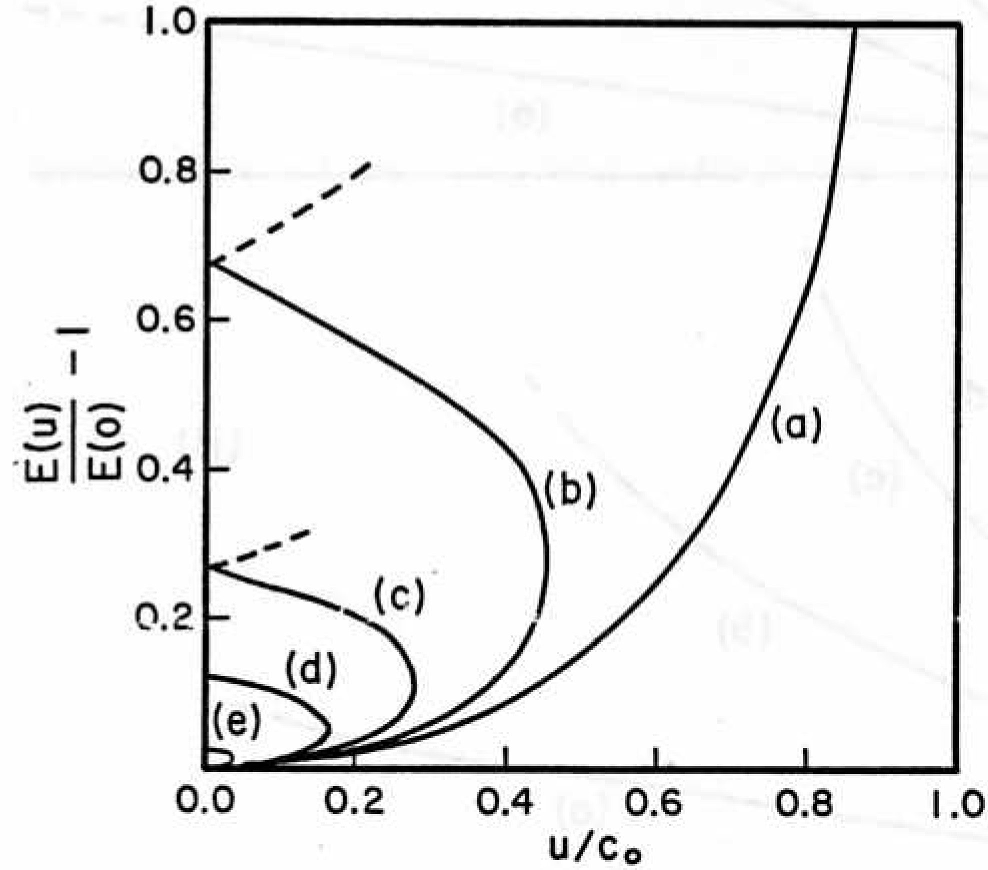


FIG 1 (iii)



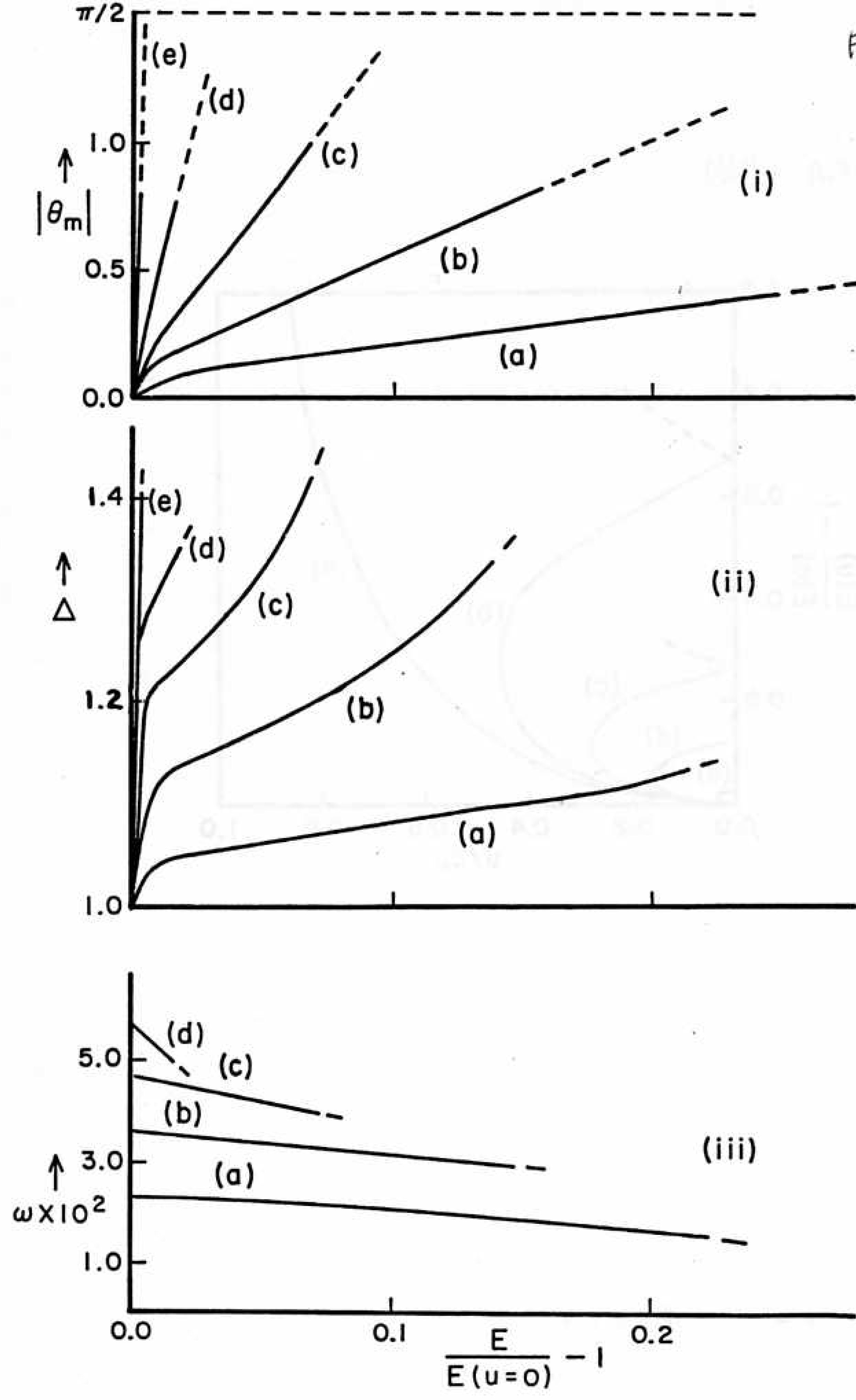


FIG 2
π/2
1.0
0.5
0.0
|θ_m|
(a)
(b)
(c)
(d)
(e)
(i)
1.4
1.2
1.0
Δ
(ii)
5.0
3.0
1.0
ω X 10^2
(iii)
0.0
0.1
0.2
E / E(u=0) − 1