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CYCLOTRON-LIKE OSCILLATIONS AND BOUNDARY EFFECTS IN THE 2-VORTEX DYNAMICS OF EASY-PLANE MAGNETS

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1 INTRODUCTION

We study the dynamics of vortices in the 2-dimensional anisotropic Heisenberg model with easy-plane symmetry (XY-symmetry). We want to find out whether the vortex trajectory $\boldsymbol{X}(t)$ follows the generalized Thiele equation

$$\boldsymbol{F} + \boldsymbol{G} \times \dot{\boldsymbol{X}} = \mathbf{M}\ddot{\boldsymbol{X}} \tag{1}$$

which was derived in another paper[1] of this volume. For a single vortex the mass tensor is $M_{ij} = M\delta_{ij}$, where i, j denote the spatial components in the XY-plane, and

$$M \approx \frac{\pi}{4\delta} \ln \frac{L}{a_0}. \tag{2}$$

δ is the anisotropy parameter in the Hamiltonian[1], L and a_0 are upper and lower cut-offs in the order of the system size and the lattice constant, respectively. The exchange constant J and the spin length S are set equal to one.

$\boldsymbol{F}$ is a force, either external or from another vortex, the gyrotropic force $\boldsymbol{G} \times \dot{\boldsymbol{X}}$ plays formally the same role as the Lorentz force on an electric charge. However, the gyrovector $\boldsymbol{G}$ acts like a kind of self-induced magnetic field, for a single vortex

$$\boldsymbol{G} = 2\pi pq\boldsymbol{e}_z. \tag{3}$$

Here $q = \pm 1$ is the vorticity, p is the out-of-plane spin component at the vortex center. In the continuum limit $p = 0, \pm 1$, therefore p is the polarization of the out-of-plane structure of the vortex.

In this paper we want to calculate the effects of the vortex mass M on the path $\boldsymbol{X}(t)$ and test the results by computer simulations for the original spin system, i.e. by integrating the Landau-Lifshitz equation, from which eqn. (1) was derived.

For zero force, eqn. (1) simply predicts a rotation with the cyclotron frequency

$$\omega_c = \frac{G}{M}. \tag{4}$$

However, for a simulation this case is particularly difficult because it requires a non-zero initial velocity. This is a problem because the Landau-Lifshitz equation is of first order in time, therefore the initial spin configuration must be chosen such that it represents already a moving vortex. Unfortunately only static vortex solutions are known, a moving vortex is deformed in a way which is known only in the continuum limit and only far away from the center[1]. This problem will be treated in detail in a paper which is in progress.

For this reason a force is needed to make a vortex move which is initially at rest. For the simplest case of a constant force $\boldsymbol{F}_\circ$ we get a cycloid with the cyclotron frequency ω_c. Unfortunately it is very difficult to apply an external force to a vortex without complicating the situation (e.g., an in-plane magnetic field creates a double domain wall which connects the vortex with a boundary). Therefore we proceed here to the case of 2 vortices, where the force is $2\pi pq/r_{12}$ at a distance r_{12}. This well-known result holds only if the static out-of-plane structures of the vortices do not overlap. Therefore in our simulations we always consider situations with $r_{12} \gg r_V$ = vortex radius[1]; we choose $\delta = 0.1$ for which $r_V = 1.5$ lattice constants. We must consider two out-of-plane vortices ($p_1, p_2 = \pm 1$) in order to have long observation times, two in-plane vortices ($p_1 = p_2 = 0$) would either annihilate after a short while, or stop due to the pinning forces of the lattice.

2 STATIONARY AND NON-STATIONARY 2-VORTEX SOLUTIONS

Stationary solutions of (1) are defined as motions where both vortices have constant speed V_0. There are 4 physically different scenarios, 2 rotational and 2 translational ones: if the gyrovectors are parallel ($p_1 q_1 = p_2 q_2$), the 2 vortices rotate around each other; i.e. they both rotate at a radius R_0 around their center of mass, where the angular frequency is

$$\omega_0 = \frac{\omega_c}{2} - \frac{\omega_c}{2}\sqrt{1 - \frac{q_1 q_2}{\pi {p_1}^2}\frac{M}{{R_0}^2}}. \tag{5}$$

For $M/{R_0}^2 \ll 1$ an expansion gives

$$\omega_0 {R_0}^2 = \frac{q_2}{2p_1}\left(1 + \frac{q_1 q_2}{4\pi {p_1}^2}\frac{M}{{R_0}^2}\right). \tag{6}$$

This predicts that a vortex-vortex (V-V) pair rotates faster than a vortex-antivortex (V-AV) pair.

If the gyrovectors are antiparallel, both vortices perform a parallel translational motion at a distance $2R_0$, where

$$V_0 R_0 = \frac{q_2}{2p_1}. \tag{7}$$

The simplest non-stationary solutions are obtained from an expansion around the stationary solutions. We set $\boldsymbol{X} = \boldsymbol{R}_\circ + \boldsymbol{x}$ with $|\dot{\boldsymbol{R}}_\circ| = V_0$ and expand $\boldsymbol{F} = \boldsymbol{F}_\circ + \Delta\boldsymbol{F}$,

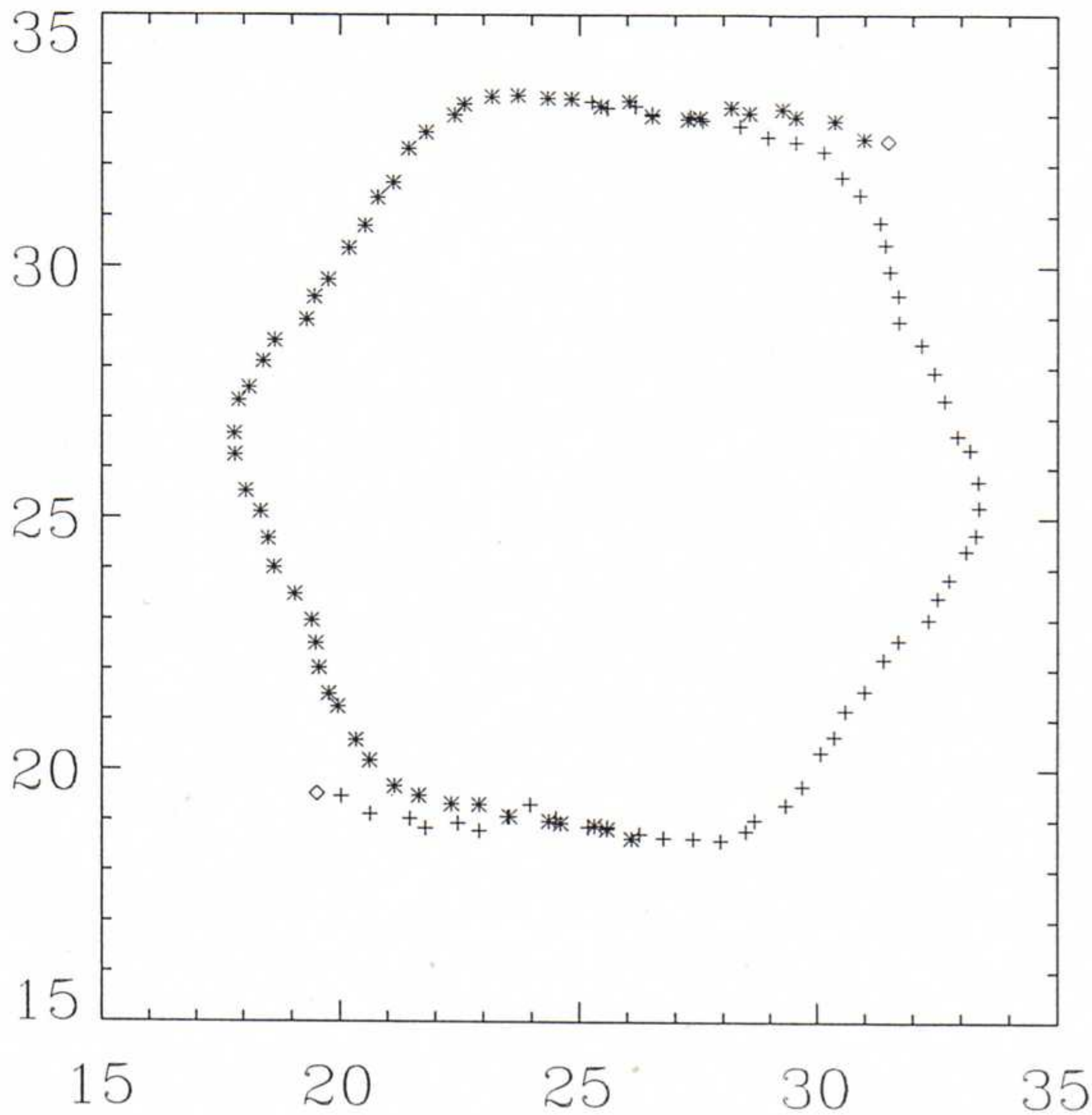

Figure 1: *Trajectories of a vortex and an antivortex with parallel gyrovectors,* 50×50 *square lattice,* $\diamond$*: initial positions, a small damping during the first 40 integration steps removes spin waves which have been excited initially.*

where $\Delta \boldsymbol{F}$ is linear in $\boldsymbol{x}$. We linearize the equations for $\boldsymbol{x}(t)$ and obtain an elliptic motion with the frequency

$$\omega = \omega_c \sqrt{1 - \frac{q_1 q_2}{2\pi {p_1}^2} \frac{M}{R_0^2}}. \tag{8}$$

This holds for the rotational cases, for the translational ones the sign in the square root is changed and 2π is replaced by 4π.

Our result means that we predict (elliptic) cyclotron-like oscillations around the stationary solutions for all four scenarios; however, for V-V rotation and V-AV translation R_0 must be larger than a certain minimum value (otherwise the square root in (8) would become imaginary).

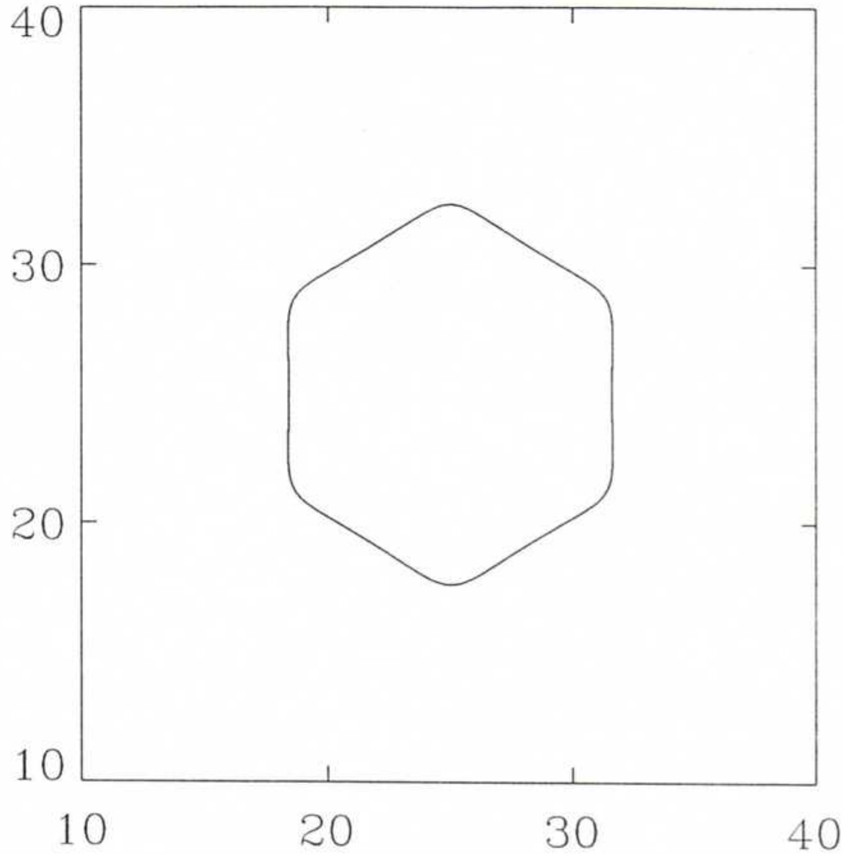

Figure 2: *Numerical solution of the generalized Thiele equation (1) for a vortex interacting with an antivortex at $-\vec{X}$. Parameters and initial conditions from the data for Fig. 1.*

3 COMPUTER SIMULATIONS

As initial condition we take a linear superposition of 2 single planar vortex solutions because the Laplace equation holds for the in-plane angle field. For the vortex centers we choose symmetric positions on a square lattice. In order to obtain definite polarizations $p_{1/2}$ we give small out-of-plane components to the 4 spins close to the center of each vortex. The Landau-Lifshitz equations are integrated using free boundary conditions, and the vortices very soon develop their full out-of-plane structure (details in ref. 2). During this process they radiate spin waves which are damped out by the inclusion of a small Landau-Gilbert damping (see ref. 2). When the damping is switched off, the vortices no longer loose energy and can be observed for arbitrary long times. Typical examples for the 4 scenarios are given in the Figs. 1,3 and also in Fig. 2 of ref. 3.

However, the various predictions of section 2 are fulfilled only for a part of the 4 scenarios:

(1) Only for V-AV rotation and V-V translation there are oscillations around the stationary solutions and the mass can be calculated from (8). In the other 2 scenarios possibly the mass is so small that the frequency of the oscillations is too high to be seen among the fluctuations due to discreteness effects.

(2) The measured values for $\omega_0 R_0{}^2$ and $V_0 R_0$ agree with the predictions only for V-V rotation. In the other scenarios there is a discrepancy which amounts up to 50 percent; this problem is cleared up in the next section.

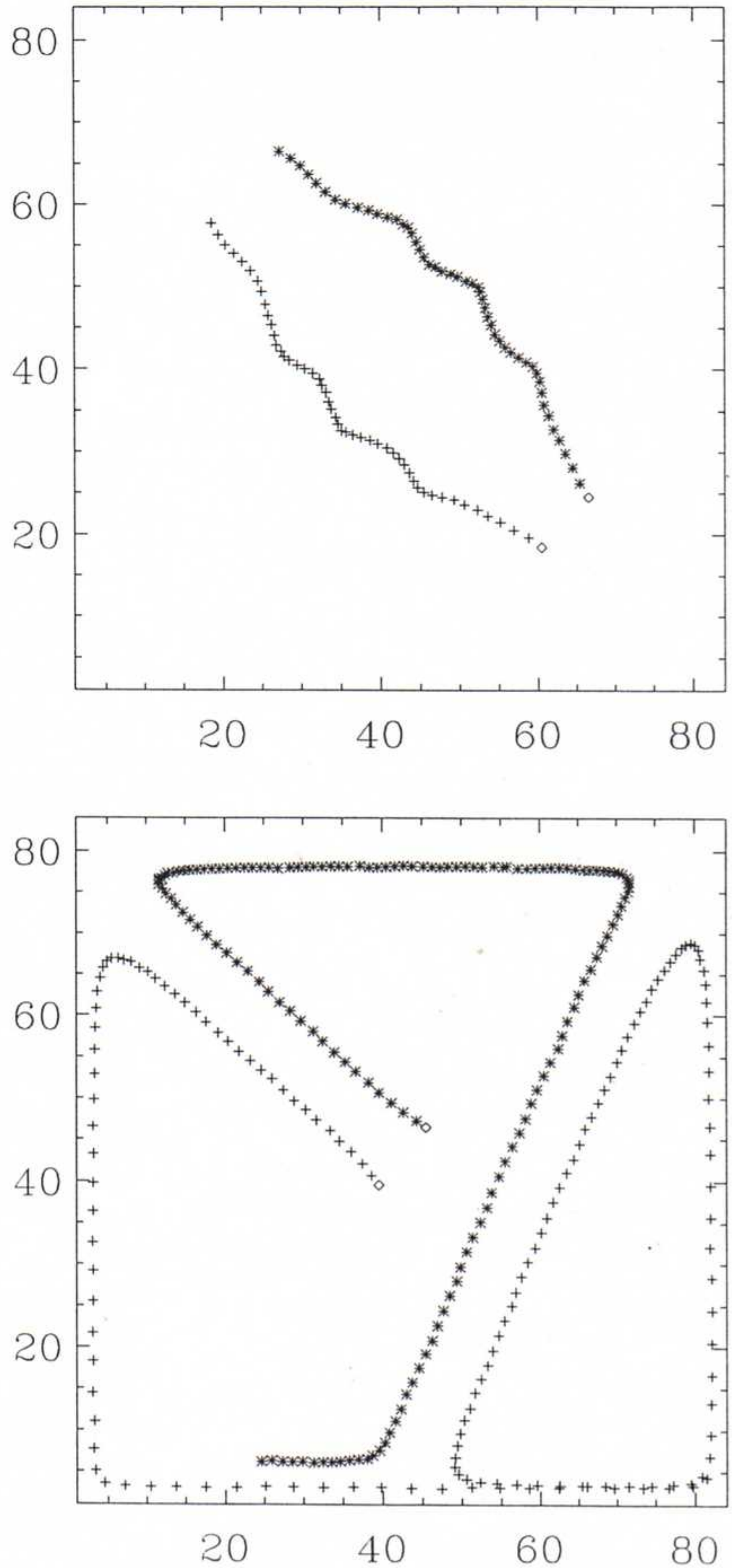

Figure 3: *Simulations for 2 vortices with antiparallel gyrovectors, ◇: initial positions. a) vortex-vortex, b) vortex-antivortex.*

In some cases, like in Fig. 1, the oscillations appear on a scale comparable to the system size. In order to make sure that this has nothing to do with the square boundaries (and in order to test our analytical approximation of section 2), we have solved numerically the generalized Thiele eqn. (1) for a vortex interacting with an antivortex at $-\boldsymbol{X}$. Here the initial conditions for the position and velocity were taken from the data for Fig. 1, at a time where the vortices had maximum distance. The mass was determined by (8), where ω had been measured in the simulations. Fig. 2 shows very clearly that the hexagon-like structure of Fig. 1 is not a boundary effect but can be explained by superimposing an elliptic motion on a circular one. Naturally, a closed orbit as in Fig. 2 occurs only for special initial conditions.

4 BOUNDARY AND DISCRETENESS EFFECTS

Fig. 3b demonstrates that a vortex which is close to a boundary (and far away from the other vortex) performs a motion parallel to the boundary; here $V_0 R_0 \approx 0.5$ where R_0 is the distance to the boundary. Obviously the vortex is driven by the interaction with an image vortex. A comparison with the scenarios of V-V and V-AV translation shows that the image vortex must have opposite vorticity but the same polarization.

If a vortex is not close to a boundary, its motion is dominated by the force from the other vortex in the system, but the force from the image vortices must also be taken into account. Let us consider the 2 rotational scenarios: here a simple qualitative consideration shows that in the V-V case the resulting force is enlarged; this means that $|\omega_0 {R_0}^2|$ should be larger than the value given by (5) or (6). Analogously $|\omega_0 {R_0}^2|$ should become smaller in the V-AV case. However, only the latter prediction is fulfilled in the simulations.

Obviously a quantitative analysis is necessary. In order to simplify the calculation we replace the square system by a circle. This is a good approximation if the ratio $\eta = R_0/L \ll 1$, where L is the radius of the system. A single vortex with radial coordinate R has an image at a radius L/R^2 as in 2-d electrostatics. We take such an image vortex for each of the two vortices in our system and obtain after a short calculation the following corrections: in F_0 (zeroth order term of the force), and thus also in eqn. (5), the product $q_1 q_2$ is replaced by

$$Q_0 = q_1 q_2 + 4\eta^l + O(\eta^{l+4}). \tag{9}$$

Here $l = 4$ for the V-V case, but $l = 2$ for the V-AV case. This result means that in the latter case $|\omega_0 {R_0}^2|$ is reduced, compared with eqn. (5),(6); and in fact this is observed. In the former case the correction is two orders smaller and therefore could not be observed.

The image vortices naturally change also the first order term ΔF. After a lengthy calculation we obtain that $q_1 q_2$ in eqn. (8) is replaced by

$$Q = q_1 q_2 + b\eta^l + O(\eta^{l+4}). \tag{10}$$

where $b = 12$ for V-V and $b = 8$ for V-AV, the powers l are the same as above.

Before we can eventually evaluate quantitatively the simulation data, we must also include discreteness effects. Here the most important effect seems to be a reduction of the polarization $|p|$ below the continuum limit value 1. Therefore we consider $|p|$ to be unknown, in addition to the mass M. The 2 unknowns can be determined by the 2 equations (5) and (8), because ω_0, R_0 and ω are measured; $q_{1/2}$, $\mathrm{sgn}(p_{1/2})$ and the system size L are input data.

The above analysis refers to the 2 rotational scenarios. For the 2 translational ones we consider a sufficiently long rectangle with width $2L$ and in a first approximation we put 2 images at the same distance from the border as the real vortices. This results in a replacement of (7) by $V_0 R_0 = Q_0/(2p_1 q_1)$ with

$$Q_0 = q_1 q_2 + b\eta^l + O(\eta^{l+1}) \tag{11}$$

where $b = 1, l = 2$ for V-V and $b = 2, l = 1$ for V-AV. Thus $V_0 R_0$ is increased in the former case, while it is decreased in the latter case; in fact both effects are observed in the simulations. Q_1 also gets a different form, compared to (10), but this is not displayed here because it will not be used in the following.

5 DATA ANALYSIS AND CONCLUSIONS

Let us consider the V-AV rotation because here ω can be measured more accurately than in the V-V translation where the observation time is rather short. We obtain $|p| \approx 0.85$ (except for small radii R_0 where $|p|$ gets even smaller). This value agrees well with an estimate from the vortex configuration itself (e.g., one can take a weighted average of the out-of-plane components at the four lattice points closest to the vortex center).

However, the values for the mass M do not agree at all with the estimate (2): (a) M is about one order of magnitude larger; (b) M is not a constant, it increases considerably for smaller R_0; (c) the size dependence may be roughly linear, instead of logarithmic. However, our largest lattice was 178×178, which is too small to determine conclusively the size dependence.

The points (a) and (b) indicate that the measured quantity M is some kind of effective mass of a 2-vortex system, and not the single-particle mass (2). This idea is strongly supported by the fact that our theory so far cannot explain why no cyclotron oscillations are seen in the V-V rotation and V-AV translation. This means that the masses of these 2-vortex systems are probably small, while the masses of the 2 other systems (V-AV rotation and V-V translation) are large.

Different masses for different 2-vortex systems are possible if there are 2-vortex contributions to the mass. They would appear on the rhs of (1):

$$\boldsymbol{F} + \boldsymbol{G} \times \dot{\boldsymbol{X}}^{\alpha} = \mathrm{M}^{\alpha\alpha}\ddot{\boldsymbol{X}}^{\alpha} + \mathrm{M}^{\alpha\beta}\ddot{\boldsymbol{X}}^{\beta} \tag{12}$$

Here α and β denote the first and the second vortex, resp.; $M_{ij}^{\alpha\alpha} = M\delta_{ij}$ as before. Because of the symmetry of our 4 scenarios $\boldsymbol{X}^{\beta}$ can be expressed by $\boldsymbol{X}^{\alpha}$: for the rotational cases $\boldsymbol{X}^{\beta} = -\boldsymbol{X}^{\alpha}$, and for translational motion, e.g. in X_1-direction, $X_1^{\beta} = X_1^{\alpha}$ and $X_2^{\beta} = -X_2^{\alpha}$.

Therefore the rhs of (12) can be written as $\mathrm{M}^{tot}\ddot{\boldsymbol{X}}^{\alpha}$ and we have again a generalized Thiele equation of the form (1). However, the mass tensor is no longer proportional to the unit. We expect a diagonal matrix if we choose a coordinate system with axes parallel and perpendicular to the direction of motion.

Naturally these ideas would not help much, if $\mathrm{M}^{\alpha\beta}$ is not known. But in fact, in another paper[3] of this volume an equation like (12) is derived from the Landau-Lifshitz equation and $\mathrm{M}^{\alpha\beta}$ can be calculated explicitly. This equation is more complicated than (12), because there are also 2-vortex contributions to the gyrovector.

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