Electrodynamics II Exam 3. Part A (120 pts.) Closed Book Radiation from Acceleration

Name____ KSU 2016/05/10 14:00 - 15:50

Instructions: Some small derivations here, state your responses clearly, define your variables in words if they are not common usage. Electron constants: $e = 1.602 \times 10^{-19}$ C $= 4.80 \times 10^{-10}$ esu, $mc^2 = 0.511$ MeV, $r_0 = \frac{e^2}{mc^2} = 2.82$ fm.

1. (16) A point charge e moves with velocity $\mathbf{v}(t)$ on a trajectory $\mathbf{r}(t)$, where t is the time in some lab frame.

a) (8) Write formulas that give its instantaneous charge density $\rho(\mathbf{x}, t)$ and current density $\mathbf{J}(\mathbf{x}, t)$ at point $\mathbf{x}$.

b) (8) The path in space-time can be written as a 4-vector $r^\alpha(\tau) = (r^0(\tau), \mathbf{r}(\tau))$, that evolves with the proper time τ. How can the results for ρ and $\mathbf{J}$ be written as one integral formula for the 4-current $J^\alpha(x)$, at observer's space-time point $x = (ct, \mathbf{x})$?

2. (16) In the Lorenz gauge, $\partial_\alpha A^\alpha = 0$, the wave equation from Maxwell's equations is $\partial_\beta \partial^\beta A^\alpha(x) = \frac{4\pi}{c} J^\alpha(x)$.

a) (8) Changing the RHS to $\delta^{(4)}(x-x')$, the equation gives the Green's functions for the wave operator. Write an expression for the retarded one, $G_r(x-x')$, that explicitly shows its depedence on the distance $R = |\mathbf{x}-\mathbf{x}'|$.

b) (8) Also write a fully covariant expression for $G_r(x-x')$, where R does not explicitly appear.

3. (8) Explain the *light cone condition* for radiation, in words and in an equation.

4. (8) Write the *general integral expression* for the solution $A^\alpha(x)$ of Maxwell's equations when the 4-current $J^\alpha(x)$ is given, using the retarded Green function.

5. (12) Using the integral expression for 4-current of a moving charge (# 2), show how to use the retarded Green function to get the Liénard-Wiechert potentials $A^\alpha(x)$ expressed in covariant form. (Show the steps of evaluating any needed integrations.)

6. (18) A charge is moving on some path *non-relativistically* with time-varying scaled velocity $\boldsymbol{\beta}(t) = \mathbf{v}(t)/c$.

a) (6) Write a formula for the part of the electric field that contributes to radiation.

b) (6) Write a formula for the distribution of instantaneous radiated power per unit solid angle, $dP/d\Omega$.

c) (6) Write a formula for the total instantaneous power radiated in all directions.

7. (8) A charge is moving on some path *relativistically* with a given 4-momentum $p(\tau)$. What is the covariant formula for the total instantaneous radiated power (generalize the result of problem 6c)?

8. (8) For a charge in arbitrary relativistic motion with scaled velocity $\boldsymbol{\beta}(t)$, what is a formula for the part of its electric field that produces radiated power?

9. (8) A charge is accelerated relativistically along a straight line. At some instant it is moving with a large value of $\gamma = (1 - \boldsymbol{\beta}^2)^{-1/2}$ and some $\dot{\boldsymbol{\beta}}$. In terms of these variables, approximately how large is the full angular width of its radiation pattern?

10. (18) An energtic charge undergoes relativistic cyclotron motion at frequency ω_0 and radius ρ. Its radiation beam sweeps across an observer in the plane of motion.

a) (6) What is the approximate spatial length of each observed individual light pulse?

b) (6) What is the lower limit of frequencies present in the radiation spectrum?

c) (6) What is the upper limit of frequencies present in the radiation spectrum?

Part A Score = ___________/120

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Instructions: Please show the details of your derivations. Explain your reasoning for full credit. Open-book only. Do both problems.

1. (68) Recall the covariant formula for the instantaneous power radiated by an accelerated charge:

$$P = -\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right). \tag{1}$$

a) (10) Express the τ-derivative of the 3-momentum $\mathbf{p}$ in terms of the applied force.

b) (10) Express the τ-derivative of the scaled energy $p_0 = E/c$ also in terms of the applied force.

c) (10) Combine your results to get the instantaneous radiated power in terms of the force, for an arbitrary case.

d) (18) Now consider an electron linearly accelerated from rest up to a final total energy of 50.0 MeV by a uniform electric field of strength $E_0 = 100.$ MV/m. Discuss how the instantaneous radiated power changes as the electron moves from its initial to final position. Give the numerical values of instantaneous power being radiated when the energy is 0.511 Mev and also when it is 50.0 MeV.

e) (20) Over the entire duration of the electron's acceleration, what total energy was radiated? Give a formula and find a numerical value.

2. (62) A synchrotron is injected with an electron of energy 50.0 Mev. The radius of the circular motion is 1.00 m. Power is continuously supplied to the machine to keep the electron at a constant energy.

a) (8) Calculate the fundamental frequency ω_0 of the orbit in rad/sec.

b) (10) Estimate the time duration in seconds of each pulse of radiation as seen by an observer in the plane of the orbit.

c) (16) Over what range of wavelengths (give numerial values) does the electron emit appreciable radiation?

d) (16) How much power in watts must be supplied to maintain the 1.00 m radius orbit?

e) (12) If the electron at energy 50.0 MeV is substituted by a proton (mc^2 = 938 MeV) of 50.0 MeV kinetic energy, how is the power in d) modified?

Part B Score = ____________/130