Electrodynamics II   Exam 2. Part A (130 pts.) Closed Book   Relativistic Electrodynamics

Name____   KSU 2016/04/15

Instructions: Use CGS-Gaussian units. No derivations here, just state your responses clearly, and define your variables in words. Write on other side if needed.

1. (8) Make a concise statement of the *principle of relativity* (the first of two postulates used by Albert Einstein in 1905):

2. (10) Inertial frame K' moves with velocity $\boldsymbol{\beta} = \beta\hat{z}$ with respect to inertial frame K. The coordinate axes of the two frames are parallel. Write out the Lorentz transformation that gives $t'x'y'z'$ in terms of $txyz$.

3. (8) A rod of length L_0 along the $\hat{z}$ direction is at rest in frame K' of the previous question. Show how to use that transformation to get its length as measured in frame K.

4. (8) Besides any space-time point $x = (ct, \mathbf{x})$, give two other examples of quantities that transform as 4-vectors. Show their time and space components (symbols) and state their meaning also in words.

5. (8) An object moves in some reference frame K with a variable velocity $\mathbf{v}(t) = c\boldsymbol{\beta}(t)$. Write an expression giving the proper time change $\Delta\tau$ when the time in K evolves from t_1 to t_2.

6. (6) A particle of rest mass m has a 4-momentum $p = (E/c, \mathbf{p})$. With correct factors of c, what is the squared length $p \cdot p$ of this 4-vector?

7. (8) A particle has energy E and 3-momentum $\mathbf{p} = p_z \hat{z}$ as measured in frame K. Another inertial frame K' moves with respect to K at relative velocity $\boldsymbol{\beta} = \beta \hat{z}$. What is the particle's energy in frame K'?

8. (8) An arbitrary linear transformation between inertial frames K and K' can be written for 4-vectors in the form $x' = \mathsf{A} x$, where A is a 4×4 matrix.

a) (4) For arbitrary Lorentz transformations, what condition must A satisfy?

b) (4) What is the definition of a "proper Lorentz transformation"?

9. (12) A proper Lorentz transformation written as $x' = \mathsf{A} x$ has a matrix expressed as $\mathsf{A} = e^{\mathsf{L}}$, where L is a traceless 4×4 real matrix. For the case of a boost at rapidity ζ along the $\hat{z}$ axis,

a) (4) Write out the matrix A.

b) (4) Write out the matrix L.

c) (4) Write out the matrix that is the generator of infinitesimal boosts along the $\hat{z}$ axis.

10. (10) Give the definition of the electromagnetic field tensor $F^{\alpha\beta}$. How do you get the electric field components from it?

11. (8) In terms of the EM field tensor $F^{\alpha\beta}$, how do you write the covariant form of equation of motion for the 4-velocity U^α of a charged particle exposed to that field?

12. (8) Write out a relativistic Lagrangian L for a particle of mass m, charge e, interacting with a given electromagnetic field described by 4-potential A^α.

13. (6) If a particle of mass m, charge e has mechanical (or kinetic) momentum $\mathbf{p}$, how do you express its canonical momentum $\mathbf{P}$ when exposed to EM fields?

14. (12) Based on the electromagnetic field tensor $F^{\alpha\beta}$ and the 4-current J^α,

a) (6) Write out a Lagrangian density $\mathcal{L}$ for electromagnetic fields produced by arbitrary J^α.

b) (6) What are the associated inhomogeneous Maxwell's equations in their covariant form?

15. (10) The EM field tensor $F^{\alpha\beta}$ and its dual $\mathcal{F}^{\alpha\beta}$ can be used to make two quantities that are invariant under proper Lorentz transformations. What are they? Also express them in terms of electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$.

Part A Score = ___________/130

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Instructions: Please show the details of your derivations here. Explain your reasoning for full credit. Open-book only. Do both problems.

1. (50) A moving electron (rest mass energy $mc^2 = 0.511$ MeV) collides with a positron initially at rest in the lab. The collision converts the electron-positron pair into a muon-antimuon ($\mu\bar{\mu}$) pair, with rest mass energies of 140 MeV each. Ignore any neutrinos that may be required to satisfy all conservation laws of particle physics.

a) (10) How large is the threshold total energy required for this process, as measured in the center of momentum frame?

b) (20) Calculate the energy E_1 (in MeV) of the incident electron at threshold, in the lab frame.

c) (20) At threshold, how fast is the center of momentum frame moving with respect to the lab?

2. (70) An electron starts from rest (in the lab frame) at time $t = 0$ in a region of uniform electric field $\mathbf{E} = E_0 \hat{x}$. There is no magnetic field.

a) (20) Write the equations of motion for the components (U^0, U^1) of the 4-velocity, considered as functions of the proper time τ.

b) (20) Solve these equations for the given initial conditions, evaluating all the constants of integration.

c) (15) The electron accelerates until it reaches an energy $E = 5mc^2$. How long did this take, measured in the lab frame?

d) (15) How far did the electron travel to attain the energy $E = 5mc^2$.

Part B Score = ____________/120