Electrodynamics II Exam 2. Part A (130 pts.) Closed Book Scattering, Special Relativity

Name______________________________________ KSU 2015/04/13 8:00 a.m.

You may use SI units for scattering questions and CGS units for relativity topics. No derivations here, just state your responses clearly, and define your variables in words.

1. (8) In scattering problems, the main goal is to find the differential scattering cross section, $d\sigma/d\Omega$. How do you calculate this? Give a formula and explain.

2. (6) What is the definition of scattering in the Rayleigh limit?

3. (6) State physically why it helps to use the Rayleigh limit.

4. (8) In the Rayleigh limit, how does the total scattering cross section of unpolarized light from a small dielectric sphere depend on the wavelength and the radius of the sphere?

5. (8) Unpolarized monochromatic light scatters from a small dielectric sphere. At what direction relative to the incident beam will the total scattered intensity be the least? Explain.

6. (8) Monochromatic light polarized perpendicular to the scattering plane (defined by $\vec{k}$ and $\vec{k}'$) scatters from a small dielectric sphere. At what direction relative to the incident beam will the total scattered intensity be the least? Explain.

7. (8) Give a statement of the Postulate of Relativity (as assumed by Einstein, Poincaré, etc.).

8. (8) Give a statement of the other Postulate that Albert Einstein used in the development of the Special Theory of Relativity.

9. (8) Two events viewed in an inertial reference frame K occur at space-time points $(ct_a, \vec{x}_a)$ and $(ct_b, \vec{x}_b)$. If the *same two events* are viewed in another inertial frame K' in motion relative to K, what quantity is invariant in the two frames?

10. (8) For a Lorentz boost along the x^1 direction, where the K' frame moves at speed $v = \beta c$ relative to K, write out the 4×4 transformation matrix that gives K coordinates (x^0, x^1, x^2, x^3) in terms of K' coordinates (x'^0, x'^1, x'^2, x'^3) (using the contravariant coordinates here and rapidity ζ).

11. (8) A particle (or a clock) is moving with some varying velocity $\beta(t)$ as viewed in a lab frame K from an initial time t_a to final time t_b. How do you write an expression for the time interval in the reference frame of the particle (or, the time elapsed on the moving clock).

12. (6) The energy-momentum 4-vector is $p^\alpha = (E/c, \vec{p})$. What does its invariant squared length depend on?

13. (8) The contravariant components of space-time 4-vector $x^\alpha = (ct, x^1, x^2, x^3)$ in frame K undergo a Lorentz transformation to a moving (K') frame by $x'^\alpha = \frac{\partial x'^\alpha}{\partial x^\beta} x^\beta$. Write an equation that expresses how the 4-momentum p^α is transformed to the K' frame.

14. (8) Consider an infinitesimal Lorentz boost along the x^1 direction with rapidity $\zeta \ll 1$. Write out the matrix K_1 that is the generator of these boosts.

15. (8) If you have the three generators of boosts along the Cartesian axes, K_1, K_2, K_3, how do you write the formal operator $A_{\text{boost}}(\vec{\beta})$ that produces a pure Lorentz boost with velocity $\vec{v} = \vec{\beta} c$?

16. (8) Write a formal definition of the components of the electromagnetic field tensor $F^{\alpha\beta}$ in terms of the 4-potential A^α.

17. (8) Write the equation for electromagnetic or Lorentz "4-force" on a charge q in covariant form. Hint: You need to use $F^{\alpha\beta}$ and produce a force-like quantity that transforms as a 4-vector.

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Please show the details of your derivations here. Explain your reasoning for full credit. Open-book and 2-page note summary allowed.

1. The threshold kinetic energy $T_{\rm th}$ in the laboratory for a given reaction is the kinetic energy of the incident particle on a stationary target just sufficient to make the center of mass energy W equal to the sum of the rest energies in the final state.

   Consider a pi-meson photoproduction reaction (photon γ incident on a stationary proton),

$$\gamma\, p \longrightarrow \pi^0\, p.$$

   The rest energies are $m_p = 938.5$ MeV for the proton and $m_\pi = 135.0$ Mev for the pion ($c = 1$ units).

   a) (10) What minmum total energy W must be present in the "center of mass" frame for this reaction to be possible? (This shouldn't require much calculation.)

   b) (25) Use conservation of 4-momentum and invariant scalar products to determine the threshold photon energy E_γ needed for this reaction, in the lab frame, in MeV.

   c) (25) Using a Lorentz transformation (or other method), find the velocity β of the center of mass frame when the photon has the threshold energy.

   d) (Bonus, 20) How large is the final state proton energy, if this "collision" could be considered as head-on, with momentum components only along the photon direction?

2. In an inertial reference frame K there are static and uniform electric field E_0 parallel to the x-axis and magnetic induction $B_0 = 2E_0$ in the xy-plane at angle θ to the x-axis.

   a) (25) Make a Lorentz boost of these fields with velocity $\vec{\beta} = \beta\hat{z}$ along the z-axis, via Jackson Eq. (11.149):

$$\vec{E}' = \gamma(\vec{E} + \vec{\beta} \times \vec{B}) - \frac{\gamma^2}{\gamma + 1}\vec{\beta}(\vec{\beta} \cdot \vec{E})$$

$$\vec{B}' = \gamma(\vec{B} - \vec{\beta} \times \vec{E}) - \frac{\gamma^2}{\gamma + 1}\vec{\beta}(\vec{\beta} \cdot \vec{B})$$

   Give the results for all field components in the K' frame.

   b) (25) Look for a reference frame K' where $\vec{E}'$ and $\vec{B}'$ are parallel. Determine the needed boost velocity $\vec{\beta}$, by forcing $\vec{E}' \times \vec{B}'$ to be zero. You should obtain β as a function of θ in the original frame.

   c) (10) In the limit $\theta = \pi/2$, what boost speed β is needed to make $\vec{E}'$ and $\vec{B}'$ parallel?