Name____ KSU 2015/03/02

Instructions: Use SI units. No derivations here, just state your responses clearly, and define your variables in words. For the waveguide questions, assume it to be filled with a medium with parameters ϵ, μ, and the axis is the z-axis.

1. (10) For a waveguide made from a very good conductor, which components ($\perp$ or $\|$) of **E** and **H** are nearly zero at the surfaces of the conductor?

2. (10) Write out the differential equation that any component of the fields should satisfy in a waveguide, for a mode propagating down the axis as $\exp(ikz)$ with wavevector k.

3. (10) For a transverse magnetic mode (TM) in a waveguide, which component of **E** or **H** acts as ψ that can be used to determine all the other field components?

4. (10) For a TM waveguide mode, what is the boundary condition on the component ψ that you answered in the previous question?

5. (10) If a certain waveguide mode has some cutoff frequency ω_λ, what does that mean?

6. (10) A waveguide mode has a cutoff frequency ω_λ. If waves at frequency ω are sent into the waveguide, with what propagation vector k do they propagate?

7. (10) A cylindrical resonant cavity with ends at $z = 0$, $z = d$ is oscillating in a TE mode. Which EM field component determines all the other components?

8. (10) For a cylindrical resonant cavity oscillating in a TE mode, what are the boundary conditions on the field component of the previous question a) on the walls b) on the ends at $z = 0$ and $z = d$?

9. (10) Give a definition of the Q of a resonant cavity. Explain what the Q means for practical purposes.

10. (10) The vector potential of an oscillating electric dipole is $\vec{A} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}(-i\omega\vec{p})$. From that, what is an expression for the magnetic field?

11. (10) Very far from an oscillating electric dipole, use your previous answer to show the polarization direction of the electric field vector.

12. (10) A radiation source produces certain fields $\vec{E}$ and $\vec{H}$ far from the source. How do you use them to obtain the power radiated per unit solid angle, along a direction $\hat{n}$?

13. (10) What is the definition of a "vector spherical harmonic?" List one orthogonality property that they have.

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Instructions: Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book.

The starting point for these problems are the fields (9.18) of an oscillating electric dipole **p**:

$$\mathbf{H} = \frac{ck^2}{4\pi}(\mathbf{n}\times\mathbf{p})\left(1-\frac{1}{ikr}\right)\frac{e^{ikr}}{r} \tag{1}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\left\{k^2(\mathbf{n}\times\mathbf{p})\times\mathbf{n}+[3\mathbf{n}(\mathbf{n}\cdot\mathbf{p})-\mathbf{p}]\left(\frac{1}{r^2}-\frac{ik}{r}\right)\right\}\frac{e^{ikr}}{r} \tag{2}$$

1. A thin rod of length $2a$ has a uniform distribution of positive charge $+e$ on one half and negative charge $-e$ on the other half, forming an electric dipole. It lies in the xy plane and is set rotating at angular frequency ω around the z-axis.

   a) (20) Determine its time-dependent electric dipole moment **p**. Write it in complex form with an assumed dependence on $\exp(-i\omega t)$.

   b) (20) Find the formula for the time-averaged power radiated per unit solid angle, $dP/d\Omega$, far from the dipole. Give the result as a function of spherical angles (θ, ϕ) that describe some direction in space, outward from the dipole.

   c) (20) Show the calculation of the total power radiated by the dipole.

2. Consider an oscillating electric dipole **p** with fields given by (9.18) in Jackson's text.

   a) (40) Show that the dipole radiates electromagnetic angular momentum at a rate given by

   $$\frac{d\mathbf{L}}{dt} = \frac{k^3}{12\pi\epsilon_0}\mathrm{Im}\left\{\mathbf{p}^* \times \mathbf{p}\right\}. \tag{3}$$

   Hint: The angular momentum comes from more than the transverse (radiation zone) components of the fields.

   b) (20) For the rotating dipole in Problem 1, use this result to find the rate at which it radiates angular momentum. What Cartesian components are present? Discuss whether your result makes sense.