Electrodynamics II   Exam 2. Part A (152 pts.) Closed Book   Relativistic Electrodynamics

Name____   KSU 2006/04/11

Instructions: Use CGS-Gaussian units. No derivations here, just state your responses clearly, and define your variables in words.

1. (12) Make concise statements of the two Postulates applied by Albert Einstein in his 1905 development of the Special Theory of Relativity:

   1.

   2.

2. (18) Inertial frame K' moves with velocity $\vec{\beta} = \beta\hat{x}$ with respect to inertial frame K. The xyz coordinate axes of the two frames are parallel. Write out the Lorentz transformation that gives $x'y'z't'$ in terms of $xyzt$.

3. (12) An object (like a clock) moves in some reference frame K with a variable velocity $\vec{v}(t) = c\vec{\beta}(t)$. Write an expression giving the proper time change $\Delta\tau$ when the time in K evolves from t_1 to t_2.

4. (6) A particle of rest mass m has a 4-momentum $p = (E/c, \vec{\mathbf{p}})$. With correct factors of c, what is the squared "length" $p \cdot p$ of this 4-vector?

5. (6) Explain in your own words why a space-time interval $s_{12}^2 = c^2(t_1 - t_2)^2 - |\vec{x}_1 - \vec{x}_2|^2$ should be an invariant, the same in all inertial reference frames.

6. (12) Match the types of space-time intervals with their definitions.

1. $s_{12}^2 < 0$	______	a) light-like interval
2. $s_{12}^2 = 0$	______	b) time-like interval
3. $s_{12}^2 > 0$	______	c) space-like interval

7. (20) An electron ($mc^2 = 0.511$ MeV) with total energy of 10.0 MeV is made to collide with another electron initially at rest (both measured in the lab).

   a) How large is the total energy W (in MeV) in the center of momentum frame?

   b) How fast is the center of momentum frame moving with respect to the lab?

8. (18) A certain unstable elementary particle of rest energy $mc^2 = 140$ Mev has a lifetime of 2.56×10^{-8}s when at rest. If it is given a total energy of 140 GeV in the lab, what are its

   a) relativistic factor γ?

   b) lifetime in the lab?

   c) mean decay path in the lab?

9. (12) Give the definition of the electromagnetic field tensor $F^{\alpha\beta}$. How do you get the electric field components from it?

10. (8) Write out the usual elementary relativistic Lagrangian for a particle of mass m, charge e, interacting with a given electromagnetic field.

11. (12) A particle with charge e and mass m enters a region with uniform crossed electric and magnetic fields $\vec{E}$ and $\vec{B}$.

    a) What situation leads to a net drift of the particle superimposed with a spiral motion? What is the average speed of this drifting?

    b) What situation leads to a continuous acceleration of the particle in the direction of $\vec{E}$?

12. (16) Based on the electromagnetic field 4-potential A^α and the 4-current J^α,

    a) Write out a Lagrangian density $\mathcal{L}$ for the electromagnetic field.

    b) What are the associated inhomogeneous Maxwell's equations in their covariant form?

13. (Bonus=18) Frame K' is moving relative to frame K as in Question 2. Write three equations for how the components of the electric field $\vec{E}'$ are obtained from the EM field components in K.

Electrodynamics II | Exam 2. Part B (148 pts.) Open Book | Relativistic Electrodynamics

Name____ KSU 2006/04/11

Instructions: Please show the details of your derivations here. Explain your reasoning for full credit. Open-book and 1-page note summary allowed.

1. An electron starts from rest at time $t = 0$ in a region of uniform electric field $\vec{E} = E_0 \hat{z}$. There is no magnetic field.

   a) (16) Write the equations of motion for all the components of the 4-velocity, considered as functions of the proper time τ.

   b) (20) Solve these equations for the given initial conditions, evaluating all the constants of integration.

   c) (20) The electron accelerates until it reaches an energy $E = 10mc^2$. How long did this take, measured in the lab frame?

   d) (20) How far did the electron travel to attain the energy $E = 10mc^2$.

   e) (Bonus=20) For extra credit, give numerical answers to c) and d) when the electric field strength is $E_0 = 1.00$ kV/m.

2. Consider a positron ($mc^2 = 0.511$ MeV, $q = +e$) moving at constant velocity (along $\hat{x}_1$-axis) as it passes close to a neutral atom at impact parameter $b = 1.0$ μm. Jackson gives the formula (11.152) for the fields seen in the lab frame (the atom is at rest in the lab frame)

$$E_1 = E_1' = -\frac{q\gamma vt}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \tag{1}$$

$$E_2 = \gamma E_2' = \frac{q\gamma b}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \tag{2}$$

$$B_3 = \gamma\beta E_2' = \beta E_2 \tag{3}$$

a) (16) Do the "conversion" of these CGS formulas to SI units, using the basic relations, $\vec{E}_{\text{SI}} = \frac{1}{4\pi\epsilon_0}\vec{E}_{\text{CGS}}$, $\vec{B}_{\text{SI}} = \frac{\mu_0 c}{4\pi}\vec{B}_{\text{CGS}}$.

b) (20) If the positron is travelling at a low speed $v = 0.01c$, what are the peak electric and magnetic field strengths felt by the atom? Give numbers in V/m and tesla.

c) (20) If the positron instead has total energy of 511 Mev, what are the peak electric and magnetic field strengths felt by the atom? Give numbers in V/m and tesla.

d) (16) For a 511 MeV positron, over what time interval (in seconds) is the magnetic field strength appreciable?

e) (Bonus=20) For extra credit, what positron energy would be necessary to produce a peak magnetic field strength of 1.0 tesla?