Electrodynamics I Exam 3 - Part A - Closed Book KSU 2015/12/18

Name____

Instructions: Use SI units. Where appropriate, define all variables or symbols you use, in words. Try to tell about the physics involved, more than the mathematics, if possible.

1. (10) What expression gives the time-dependent electric field in terms of scalar and vector potentials?

2. (12) In the presence of any time-dependent sources $\rho(\mathbf{x},t)$ and $\mathbf{J}(\mathbf{x},t)$, what equation is obeyed by the scalar potential when using the Lorenz gauge?

3. (10) Consider a 3D wave equation, $\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi = -4\pi f(\mathbf{r},t)$, where $f(\mathbf{r},t)$ is the source that drives some waves $\Psi(\mathbf{r},t)$. Write out the retarded space- and time-dependent Green's function for this equation.

4. (12) Write out an equation for Poynting's theorem in differential form. Explain in words what each term means physically.

5. (12) How does $\vec{L}_{\rm em} = \frac{1}{c^2}\int\, d^3r\ \vec{r}\times(\vec{E}\times\vec{H})$ transform under space inversion? Time inversion?

6. (12) A plane EM wave is traveling in the x-direction in a medium with $\mu = \mu_0$ and $\epsilon = 9\epsilon_0$. With linearly polarized $\vec{E}(x,t) = E_0\hat{z}\ \exp[i(kx - \omega t)]$ write an expression for $\vec{B}(x,t)$ in this wave.

7. (12) A plane wave travels in the x-direction: $\vec{E}(\mathbf{r},t) = E_0(\hat{y} + i\hat{z})\exp[i(kx - \omega t)]$. Looking into the wave at a fixed point in space, in which direction does the electric field vector rotate (clockwise or couterclockwise)? Which circular polarization is this (right or left)?

8. (10) When light undergoes total internal reflection at an interface between two optical media with indexes n (incident side) and n' (refraction side), what is required of the incident angle θ?

9. (10) Write an expression for the dielectric function $\epsilon(\omega)$ in a plasma.

10. (10) What does $\epsilon(\omega)$ imply for EM waves of low frequency traveling in a plasma?

Part A Score = __________/110

Name______________________________________

Instructions: Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book only, no notes.

1. (48) Consider the 3D wave equation for an EM field component $\psi(\mathbf{x}, t)$,

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = -4\pi f(\mathbf{x}, t).$$

It is being driven by a source function $f(\mathbf{x}', t') = \delta(x')\delta(y')\delta(t')$, which represents a flash of a line source along the z'-axis at time $t' = 0$. You know that the retarded 3D Green's function for this wave equation is

$$G^+(\mathbf{x}, t; \mathbf{x}', t') = \frac{1}{|\mathbf{x} - \mathbf{x}'|}\,\delta\left(t' - t + \frac{|\mathbf{x} - \mathbf{x}'|}{c}\right).$$

Consider the EM response in 2D around the line source at a distance $\rho = \sqrt{x^2 + y^2}$ from that line source.

(a) (12) Write the general integral expression for the signal $\psi(\mathbf{x}, t)$ in terms of an integration over the source function $f(\mathbf{x}', t')$. Do not yet evaluate it.

(b) (12) Write out the result after only the integration over source time t' is performed.

(c) (24) Now do the integrations over source point $\mathbf{x}'$, including over z'. Determine the final signal $\psi(\mathbf{x}, t)$ as a function in the form $\psi(\rho, t)$. Is there some restriction between the observation time t and the radial distance ρ from the source?

Hint: Due to the symmetry, the integration over z' might be easier to do as an integration over source to observer distance $R = |\mathbf{x} - \mathbf{x}'|$. What is the restriction on R?

2. (44) A plane wave of intensity $I_0 = 48.0$ kW/cm^2 and some wave vector of magnitude k is traveling in the $+\hat{z}$ direction, with its electric field vector polarized along $+\hat{x}$. Jackson's Eq. (6.121),

$$\frac{d}{dt}\left(\mathbf{P}_{\text{mech}} + \mathbf{P}_{\text{field}}\right)_\alpha + \oint_S \sum_\beta (-T_{\alpha\beta})\, n_\beta \, da = 0$$

suggests that $-T_{\alpha\beta}$ is the flux (per unit area per unit time) of α component of field momentum in the β direction. The components of the Maxwell stress tensor are

$$T_{\alpha\beta} = \epsilon_0 \left[E_\alpha E_\beta + c^2 B_\alpha B_\beta - \frac{1}{2}\left(\mathbf{E}\cdot\mathbf{E} + c^2 \mathbf{B}\cdot\mathbf{B}\right)\delta_{\alpha\beta} \right].$$

(a) (16) For the given plane wave, determine how $(-T_{zz})$ is related to the z-component of the electromagnetic momentum density, $\mathbf{g} = (\mathbf{E}\times\mathbf{H})/c^2$.

(b) (16) If the given wave is incident on a perfectly conducting mirror at normal incidence, determine the radiation pressure on the mirror, in terms of I_0 and in N m^{-2}.

(c) (12) How is the answer to part (b) changed if the angle of incidence is $\theta = 60°$?

3. (48) The imaginary part of a dielectric function is known to be

$$\frac{\epsilon_I(\omega)}{\epsilon_0} = \frac{\gamma\omega}{\omega^2 + \gamma^2}.$$

(a) (16) Apply the Kramers-Kronig relations to obtain the real part of $\epsilon(\omega)$.

(b) (16) Sketch the real and imaginary parts of $\epsilon(\omega)$ vs. ω, and identify the regions of normal and anomalous dispersion.

(c) (16) Determine the complex function $\epsilon(\omega)$ and locate its poles in the complex ω-plane. Do its poles occur in the region associated with causal response?

Part B Score = __________/140