Electrodynamics I — Midterm Exam - Part A - Closed Book — KSU 2015/09/25

Name____ Score = _____________ / 120 points

Instructions: Use SI units. Where appropriate, define all variables or symbols you use, in words. Try to tell about the physics involved, more than the mathematics, if possible.

1. (10) Write Gauss' Law in integral form. State its physical significance.

2. (10) A fixed electric charge density $\rho(\mathbf{x})$ exists in space with no boundaries. Write an expression for the **electric field** that it produces at a field point $\mathbf{x}$.

3. (10) A fixed electric charge density $\rho(\mathbf{x})$ exists in space with no boundaries. Write an expression for the **electrostatic potential** that it produces at a field point $\mathbf{x}$.

4. (10) Write an expression that gives the electrostatic field energy density in vacuum.

5. (10) From a solution for electric potential $\Phi(\mathbf{x})$, how can you obtain the surface charge density σ on a conductor boundary? Let $\hat{n}$ be the unit vector pointing perpendicularly outward from the conductor.

6. (10) From the previous two answers, how do you write the force per unit area caused by the electric field on a conductor surface? Express the result as a vector.

7. (10) Write Poisson's equation for electrostatics. Explain what it is good for.

8. (10) Write a differential equation that a Green function $G(\mathbf{x}, \mathbf{x}')$ for Poisson's equation must satisfy, for *Dirichlet* boundary conditions. Include a statement of the boundary conditions.

9. (10) A problem has *Dirichlet* boundary conditions. How do you write the general solution to the Poisson equation for electrostatic potential $\Phi(\mathbf{x})$ using a Green's function?

10. (10) In an electrostatics problem with *Neumann* boundary conditions, what is the simplest allowable boundary condition on the the Green's function $G(\mathbf{x}, \mathbf{x}')$? Hint: The result must be consistent with the differential equation that G satisfies.

11. Use delta-functions to express the charge density $\rho(\mathbf{x})$ for the following charge distributions, in the indicated coordinate systems:

    a) (10) A point charge q on the z-axis at $z = c$. Use cylindrical coordinates (ρ, ϕ, z).

    b) (10) A charge Q distributed uniformly over an infinitesimally thin circular disk of radius a centered on the z-axis and lying in the plane $z = 0$. Use spherical coordinates (r, θ, ϕ).

Name____ Score = ____________ / 130 points

Instructions: Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book only, no notes.

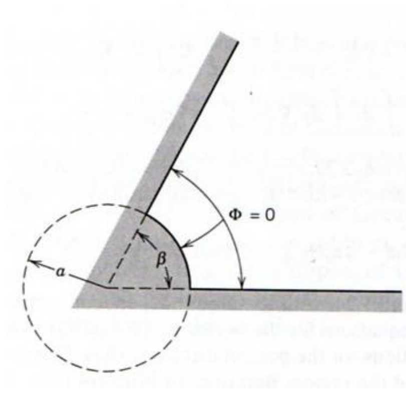
1. A point charge q is located a distance a above an infinite plane conductor held at zero potential. Use the method of images to find

   a) (20) The surface charge density on the plane, as a function of a radial coordinate ρ.

   b) (20) The force per unit area on the plane, F/A, as a function of ρ.

   c) (20) The total electric force on the plane, by integrating the force per area. Is it the result you expect?

2. A 2D region, $\rho \geq a$, $0 \leq \phi \leq \beta$ is bounded by conducting surfaces at $\phi = 0$, $\phi = 0$ and $\phi = \beta$ held at zero potential (curved 2D corner, see Fig.). The potential is determined by some charges far from the region.



a) (20) Write down a solution for the potential $\Phi(\rho, \phi)$ that satisfies the boundary conditions for finite ρ. It may have undetermined constants.

b) (20) In the case that $\beta = \pi$, the problem is that of a half cylinder on an infinite plane. To leading order, determine how the surface charge density depends on ρ along the boundary at $\phi = 0, \rho > a$.

3. For a point $\mathbf{x}$ in some volume V bounded by a surface S, Green's theorem can be arranged into an integral statement about electrostatic potential,

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\mathbf{x}')}{R} d^3x' + \frac{1}{4\pi}\oint_S \left[\frac{1}{R}\frac{\partial \Phi}{\partial n'} - \Phi \frac{\partial}{\partial n'}\left(\frac{1}{R}\right)\right] da' \tag{1}$$

where $R = |\mathbf{x} - \mathbf{x}'|$ and the other symbols have their usual meanings.

Consider proving the **mean-value theorem**: *For charge-free space the electrostatic potential at a point is its average over the surface of any sphere centered on that point.*

a) (5) To prove this theorem, first explain why the first term in Eq. 1 will be zero for the specified situation.

b) (10) Next, give a convincing mathematical argument showing why the second term in Eq. 1 will be zero for the specified situation.

c) (15) Finally, give a convincing mathematical argument showing what the value of the last term in Eq. (1) is, and explaining how this proves the theorem.