Electrodynamics I Midterm Exam - Part A - Closed Book KSU 2014/10/23

Name____

Instructions: Use SI units. Where appropriate, define all variables or symbols you use, in words. Try to tell about the physics involved, more than the mathematics, if possible.

1. (3) Write Gauss' Law in differential form. Explain the physical meaning.

2. (3) Write an expression that gives the electrostatic field energy in vacuum.

3. (3) Show how to get the capacitance of an isolated spherical conductor of radius R. How large in μF is the capacitance of the Earth ($R = 6380$ km), considered as a large conductor?

4. (3) Write a differential equation that a Green function $G(\vec{r}, \vec{r}\,')$ for Poisson's equation must satisfy, for Dirichlet boundary conditions.

5. (3) A problem has boundaries with Dirichlet boundary conditions. How do you write the solution to the Poisson equation for electrostatic potential $\Phi(\vec{r}\,)$ using a Green's function?

6. (3) Give a condition (possibly as an inequality) that identifies the limit where classical E&M theory should be replaced by quantum theory. Explain it.

7. (3) A charge density $\rho(\vec{r}\,)$ is invariant when the system is rotated through any angle around the z-axis. How can you write the general solution of Poisson's equation for the potential $\Phi(\vec{r}\,)$ in this situation?

8. (3) A linear and isotropic dielectric medium has electric susceptibility χ. How does χ enter in the formulas for the electric polarization and the electric permittivity?

9. (3) Give a formula that determines the electric dipole moment of an arbitrary but localized charge density $\rho(\vec{r}\,)$.

10. (3) If a point electric dipole $\vec{p}$ is located at position $\vec{r}_0$, what electrostatic potential does it produce at an arbitrary position $\vec{r}\,$?

11. (6) For the point dipole of the previous question, what electric field does it produce at an arbitrary position $\vec{r}$?

12. Use delta-functions to express the charge density $\rho(\vec{r})$ for the following charge distributions, in the indicated coordinate systems:

a) (3) A charge Q distributed uniformly over an infinitely thin circular ring of radius a centered on the z-axis and lying in the plane $z = b$. Use spherical coordinates (r, θ, ϕ).

b) (3) A point charge q on the x-axis at $x = x_0$. Use cylindrical coordinates (ρ, ϕ, z).

13. A point charge q is placed at a distance $d > a$ from the center of an *uncharged* isolated metal sphere of radius a.

   a) (6) Determine the electric force acting on q due to the sphere, for arbitrary $d > a$. Is it attractive or repulsive? Explain.

   b) (4) Find the asymptotic force law for $d \gg a$.

Name____

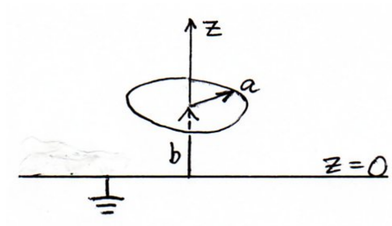
Instructions: Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book only, no notes.

1. An infinitely thin ring of total charge Q has a radius a, and is placed centered on the z-axis in the plane $z = b$, above a *grounded* infinite plane conductor at $z = 0$. The plane of the ring is parallel to the plane of the conductor.

   a) (8) Find the electric potential $\Phi(z)$ along the z-axis anywhere $z > 0$, which is the axis of the ring.

   b) (8) Expand your result of part a in power series, one valid for $z < \sqrt{a^2 + b^2}$, and another series valid for $z > \sqrt{a^2 + b^2}$.

   c) (8) Use the result of part b to find the electric potential $\Phi(r, \theta, \phi)$ for any points above the grounded plane.



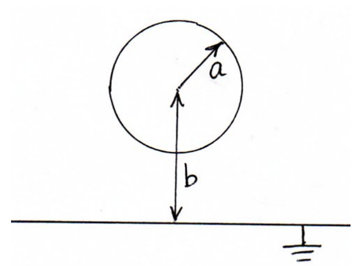
2. A very long conducting cylinder with a circular cross section of radius a is placed with its axis a distance $b > a$ away from and parallel to a grounded plane conductor. The cylinder is held at fixed potential V relative to the grounded plane.

a) (10) Use the method of images and show that by an appropriate choice of image line charges, the equipotentials are circles. Hint: The image line charge within the cylinder does not need to be along its axis.

b) (8) Find how the circle center and radius of an equipotential circle depend on a chosen value of potential Φ between 0 and V.

c) (6) Calculate the capacitance per unit length of the cylinder/plane system.

d) (6) Bonus. Find the charge density induced on either the cylinder or on the plane, as a function of angular or linear coordinate on each, respectively. (Do only one or the other.)



Useful Formulas

Legendre Polynomials

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \frac{1}{2}(3x^2 - 1), \quad P_3(x) = \frac{1}{2}(5x^3 - 3x), \quad P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3).$$

Spherical Harmonics

$$Y_{0\,0} = \sqrt{\frac{1}{4\pi}}, \quad Y_{1\,0} = \sqrt{\frac{3}{4\pi}}\cos\theta, \quad Y_{1\,\pm1} = \mp\sqrt{\frac{3}{8\pi}}e^{\pm i\varphi}\sin\theta, \quad Y_{2\,0} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1).$$

$$Y_{2\,\pm1} = \mp\sqrt{\frac{15}{8\pi}}e^{\pm i\varphi}\cos\theta\sin\theta, \quad Y_{2\,\pm2} = \sqrt{\frac{15}{32\pi}}e^{\pm 2i\varphi}\sin^2\theta, \quad Y_{l\,0} = \sqrt{\frac{2l+1}{4\pi}}P_l(\cos\theta).$$

Expansions

$$\frac{1}{|\vec{r} - \vec{r}\,'|} = \sum_{l=0}^{\infty}\frac{r_<^l}{r_>^{l+1}}P_l(\cos\gamma), \quad \frac{1}{|\vec{r} - \vec{r}\,'|} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y^*_{l\,m}(\theta', \phi')Y_{l\,m}(\theta, \phi)$$