Name____

Instructions: Use SI units. Short answers! No derivations here, just state your responses clearly.

1. (2) Write an integral expression for the magnetic induction generated in unbounded vacuum by a current density $\vec{J}(\vec{r})$.

2. (2) Now give the corresponding differential equation which the same magnetic induction must satisfy.

3. (2) What vector potential $\vec{A}(\vec{r})$ is generated by the same current distribution $\vec{J}(\vec{r})$?

4. (4) How do the scalar and vector potentials determine the fields for a time-dependent problem.

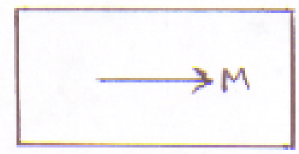
5. (2) Write out the integral form of Ampere's Law.

6. (3) Give a formula for the potential energy of a magnetic dipole when placed in an external magnetic induction $\vec{B}$.

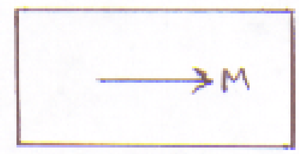
7. (4) State the boundary conditions on $\vec{B}$ and $\vec{H}$ at an interface between two linear nonconducting permeable media.

8. (2) Give the relation defining magnetic field $\vec{H}$ in terms of magnetic induction $\vec{B}$ and magnetization $\vec{M}$.

9. (2) A cylindrical permanent magnet has a uniform magnetization M along its long axis. Sketch the lines of its $\vec{B}$-field both inside and outside.



10. (2) Sketch the lines of its $\vec{H}$-field for the same permanent magnet.



11. (3) Give an expression for the magnetic dipole moment of a current distribution $\vec{J}(\vec{r})$:

12. (4) Write out Maxwell's equations for harmonic time-dependent fields:

13. (3) What quantity describes the flux of energy carried in an electromagnetic wave? Give the name and defining formula.

14. (2) How does the following quantity transform under spatial inversion?

$$\vec{L}_{\rm em} = \frac{1}{c^2} \int d^3r \; \vec{r} \times (\vec{E} \times \vec{H})$$

15. (2) How does $\vec{L}_{\rm em}$ transform under time reversal?

16. (2) A plane electromagnetic wave at angular frequency ω propagates in the z-direction in a medium with permittivity ϵ and permeability μ. Write an expression for its wavevector $\vec{k}$.

17. (3) For the same plane wave, write an expression for its electric field $\vec{E}(\vec{r}, t)$.

18. (3) Also for the same plane wave, what is the associated magnetic induction $\vec{B}(\vec{r}, t)$?

19. (2) Give a relation between the intensity I_0 in a plane EM wave and its time-averaged energy density u.

20. (2) When light undergoes total internal reflection at an interface between two optical media with indexes n (incident side) and n' (refraction side), how large is the incident angle θ?

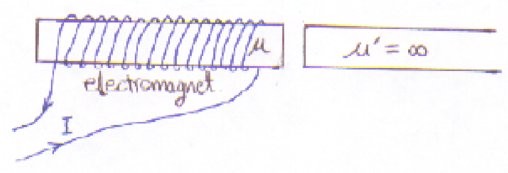
21. (3) For the same interface, at what incident angle will the reflected wave be totally polarized? And in which direction is it polarized?

22. (2) A dispersive medium has a frequency-dependent dielectric function $\epsilon(\omega)$ (dispersion). For an EM wave-packet of narrow bandwidth passing through this medium, describe at least one important effect caused by the dispersion.

Name____

Instructions: Use SI units. Please Write your derivations and final answers on these pages. Explain your reasoning for full credit. One-page note summary is allowed.

23. (16) An electromagnet is made by winding a coil with $N = 2000$ turns on a cylindrical piece of soft iron with length $l = 4.0$ cm and radius $a = 4.0$ mm, with high permeability $\mu = 400\mu_0$. One end of the electromagnet is placed a distance $d = 1.0$ mm from a long rod of the same cross section, with extremely high permeability (take $\mu' = \infty$).



Assume that the fields $\vec{B}_{\text{iron}}$ and $\vec{H}_{\text{iron}}$ within the iron are nearly uniform, and that $d \ll a \ll l$. The arrangement is surrounded by vacuum. A constant current $I = 5.0$ A is flowing in the coil.

a) (8) Apply Ampere's Law, showing the path you use, and estimate the magnitudes B_{iron} and H_{iron}, and the average magnetization M_{iron}. Give them with correct SI units.

b) (4) Determine the associated values of B_{gap} and H_{gap} in the gap between the electromagnet and the other rod.

c) (4) Estimate the force of attraction between the electromagnet and the rod, in Newtons. Does the result depend on d?

24. (12) Consider an EM plane wave propagating within a crystal of permittivity $\epsilon = 2.25\,\epsilon_0 = (9/4)\,\epsilon_0$ (for example, inside a diamond crystal), which is incident from inside on some face of the crystal. The crystal is surrounded by vacuum. Assume $\mu = \mu_0$ everywhere.

a) (8) Consider an incident angle $\theta = 60°$, which results in TIR. Take the x-axis along the boundary and the z-axis pointing out of the crystal, perpendicular to the boundary. If the incident electric field is

$$\vec{E}(\vec{r}, t) = E_0\ \hat{y}\ \exp\{i[\vec{k}\cdot\vec{r} - \omega t]\} = E_0\ \hat{y}\ \exp\{i[k(\sin\theta\ x + \cos\theta\ z) - \omega t]\},$$

write an expression for the evanescent field $\vec{E}'(x, z, t)$ on the vacuum side, explicitely displaying its dependence on x and z, and correct amplitude.

b) (4) Over what distance into the vacuum (measured in free space wavelengths) does the evanescent wave decay by a factor of e^{-1} in amplitude?

25. (16) EM waves in a plasma interact with the dielectric function,

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 - \frac{\omega_p^2}{\omega^2}.$$

a) (4) Assuming $\mu = \mu_0$, derive the dispersion relation giving $\omega(k)$ for EM waves in a plasma.

b) (4) Based on your $\omega(k)$ [or $k(\omega)$], what happens to waves with $\omega < \omega_p$, that enter this medium?

c) (4) Now suppose the plasma has $NZ = 1.0 \times 10^{22}$ electrons/m^3, and consider waves with $\omega = 3\omega_p$. What is the phase velocity v_p of the waves?

d) (4) For the same parameters as (c), what is the group velocity, $v_g = \frac{d\omega}{dk}$?

26. (10) A plane wave of intensity $I_0 = 20.0\ \text{kW/cm}^2$ is incident on a perfectly reflecting mirror at angle of incidence $\theta = 60°$. Determine the radiation pressure on the mirror in N/m^2.

27. (10) The imaginary part of a dielectric function is known to be

$$\frac{\epsilon_I(\omega)}{\epsilon_0} = \frac{\gamma\omega}{\omega^2 + \gamma^2}.$$

a) (4) Apply the Kramers-Kronig relations to obtain the real part of $\epsilon(\omega)$.

b) (4) Sketch the real and imaginary parts of $\epsilon(\omega)$ vs. ω, and identify the regions of normal and anomalous dispersion.

c) (2) Determine the complex function $\epsilon(\omega)$ and locate its poles in the complex ω-plane. Do its poles occur in the region associated with causal response?