PUPT-1535, POP-620
March 1995

'COSMOLOGICAL' MAGNETIC BRAKING AND THE FORMATION OF 'PRIMORDIAL' STARS AND BLACK HOLES

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ABSTRACT

A primeval 'seed' magnetic field of present strength $\sim 10^{-12}$G, amplified by protogalactic collapse and galactic differential rotation, could account for observed large-scale galactic magnetic fields. For primordial clouds of baryonic mass $\lesssim 10^2 M_\odot$, the magnetic braking timescale in such a field is less than the Hubble time, so the field is able to reduce angular momentum generated by tidal torques and permit the collapse of such clouds to extragalactic protostars. Such a field, amplified by collapse of a host galaxy halo of baryonic mass $\lesssim 10^{10} M_\odot$, results in a magnetic braking time less than the Hubble time for the central $\lesssim 10^8 M_\odot$ (baryonic) portion of the galaxy, and so will reduce tidal-torque generated angular momentum and form proto-black holes within high redshift hosts.

Subject headings: magnetic fields — stars: formation — quasars: general — black hole physics — galaxies: formation — cosmology: theory

1. INTRODUCTION

Protogalaxies, or other early structures, condensing under self-gravity are likely to be fairly irregular, and so would spin up their neighbours (Strömberg 1934; Hoyle 1951; Peebles 1969; Doroshkevich 1970). The resulting angular momentum builds up over time and estimates of the final mean galactic angular momentum generated by this tidal-torquing process (e.g., Ryden 1988; Warren et al. 1992; Andriani & Caimmi 1994) indicate that it agrees with what is observationally deduced for the slowly-rotating elliptical galaxies, although other predicted consequences (e.g., the large spread in the angular momentum distribution) have not yet been shown to be consistent with observations (Peebles 1995). Whatever the ultimate resolution of these issues, it is clear that, once it builds up, the angular momentum induced by tidal torques in a low redshift collapsing protoobject will result in a final object that resembles a low density, centrifugally supported, cloud. To allow low redshift collapse to a higher density compact object, the tidal-torque induced angular momentum needs to be reduced. Such compact objects have been invoked in the massive black hole and accretion disk scenario for quasars and AGN (Zel'dovich 1964; Salpeter 1964; Lynden-Bell 1969; reviewed in Rees 1984). There has also been much discussion of the consequences of extragalactic Pop III stars (e.g., Carr 1994, hereafter C94).

While there has been some discussion of the formation of extragalactic Pop III stars (e.g., Kashlinsky & Rees 1983; C94), this has been in the context of the fragmentation of a recombination epoch Jeans mass cloud (e.g., Peebles & Dicke 1968), and the angular momentum problem for low redshift formation of extragalactic Pop III stars does not seem to have been addressed. In the scenario where a quasar is an accreting massive black hole, the black hole is thought to reside in the nucleus of a virialized galactic halo. There are estimates of the number density of halos formed in the Einstein-de Sitter model (Efstathiou & Rees 1988; Katz et al. 1994), however, the limited numerical resolution of current simulations prevents an examination of black hole formation. In fact, it is unclear how the tidal-torque-generated angular momentum is reduced, to allow for the formation of a compact object at the center of the halo (e.g., Turner 1991). One way out makes use of the cosmic microwave background radiation drag on the electrons in a locally ionized region (Loeb 1993; Umemura, Loeb, & Turner 1993), which requires very early reionization through massive star formation (Susa, Sasaki, & Tanaka 1994), and is unlikely to be achieved in the adiabatic gaussian structure formation scenario. Furthermore, it is also unclear how angular momentum is reduced to allow formation of these massive stars. Another possibility is that black holes only form in those halos in the low angular momentum tail of the tidal-torque angular momentum distribution (Eisenstein & Loeb 1995). In this case black holes of mass $\sim 10^{6-7} M_\odot$ (one or two orders smaller than thought necessary, Turner 1991) would have a number density comparable to that of bright galaxies (Eisenstein & Loeb 1995). Also, there are indications that some low redshift quasars are not associated with bright galaxies (Bahcall, Kirhakos, & Schneider 1994) — it is possible that the unseen related, postulated, galaxies are at a stage prior to significant star formation, but both the low redshifts, and that it should be easier to reduce the protostellar angular momentum barrier than it is to reduce the proto-black hole angular momentum barrier, argues against this. While there are many other possible mechanisms that could reduce angular momentum, e.g., an interaction with a companion or a merger, winds, bipolar flows, shocks, internal tidal torques, effects associated with a bar or a disk or a ring or other distortion, dynamical friction, nonaxisymmetric collapse, fragmentation, etc., these have mostly been studied in the context of removing the angular momentum of the fuel needed to feed the compact object, and not in the context of compact object formation (probably because they seem to be mostly individually unlikely to suffice, and seem to be mostly ineffective during the initial 'low-density' stage of collapse; we caution that some of these processes are very difficult to model and analyze).

There is also a centrifugal barrier that must be reduced during galactic star formation. Here it is generally accepted that during the initial low-density collapse the embedded galactic magnetic field prevents the collapsing protostar from spinning up (as fast as would happen in the absence of the magnetic field) by enforcing corotation with the surrounding matter, and the resulting Alfvén waves carry the angular momentum out of the protostar (Ebert, von Hoerner, & Temesvary 1960; Mestel 1965; Mouschovias & Paleologou 1979, 1980, hereafter MP79, MP80; Dorfi 1982, hereafter D82; Nakano 1989; Tomisaka, Ikeuchi, & Nakamura 1990; reviewed in Bodenheimer 1992). In this *Letter* we examine the efficacy of magnetic braking by a postulated 'primordial' cosmological magnetic field, in alleviating the angular momentum problem for collapsing 'primordial' protostars and proto-black holes. The postrecombination residual ionization (Peebles 1993, §6) suffices to keep a cosmological magnetic field frozen to the baryonic matter.

2. MAGNETIC BRAKING TIMESCALES

We first consider primordial protostellar collapse in a FLRW cosmological model with baryon density ρ_B and with a magnetic field B, approximately uniform at least on a length scale R corresponding to a cloud of baryonic mass $M_B = 4\pi\rho_B R^3/3$. As we are interested in seeing if the angular momentum can be prevented from building up, we consider the earliest stages of collapse, and ignore the difference between the baryon density inside the cloud and that of the external material. As the collapsing cloud spins up the magnetic field lines are twisted and Alfvén waves propagate outwards along the field lines with speed

$v_A = B/\sqrt{4\pi\rho_B}$, and carry away angular momentum. The magnetic braking timescale depends on the relative orientation of the magnetic field and the angular momentum axis (e.g., MP79; MP80; D82). The simplest case is when they are parallel, and for a spherical cloud the e-folding timescale for angular momentum removal is (e.g., MP79)

$$t_A = \frac{8}{15}\frac{R}{v_A} = \frac{8}{15}\,(36\pi)^{1/6}\,\frac{{\rho_B}^{1/6}{M_B}^{1/3}}{B}. \tag{1}$$

When the magnetic field and angular momentum axis are perpendicular the braking time could be as much as an order of magnitude smaller (e.g., D82), and, more importantly, the braking timescale rapidly decreases as the cloud contracts (e.g., MP80; Basu & Mouschovias 1994), so the time for total collapse is only a little bigger than that for the first e-folding, eq. (1). Numerical simulations (e.g., D82; Tomisaka et al. 1990) show that the estimate of eq. (1) is good.

Flux conservation relates the present value B_0 of the cosmological magnetic field to that at redshift z: $B = B_0(1+z)^2$. The ratio of the angular momentum removal timescale to the timescale H^{-1} set by the Hubble parameter is

$$Ht_A = \frac{8}{15}\,(36\pi)^{1/6}\,\frac{H_0}{B_0}\,(\rho_0\Omega_B)^{1/6}\,{M_B}^{1/3}\left[\Omega_0 + \frac{\Omega_\kappa}{1+z} + \frac{\Omega_\Lambda}{(1+z)^3}\right]^{1/2}, \tag{2}$$

where H_0 and ρ_0 are the present Hubble parameter and the Einstein-de Sitter density of zero pressure matter, and Ω_B, Ω_0, Ω_κ, and Ω_Λ are the density parameters for baryonic matter, zero pressure matter (including baryons), space curvature, and the cosmological constant (Peebles 1993, §13). At sufficiently early times Ht_A is independent of redshift, as expected. With $H_0 = 100h$ km s^{-1} Mpc^{-1}, choosing cosmological parameters and formation redshift ($z > 10$) so that $h[\Omega_0 + \Omega_\kappa/(1+z) + \Omega_\Lambda/(1+z)^3]^{1/2} = 0.3$, and using the standard nucleosynthesis estimate $\Omega_B h^2 = 0.0125$, we have

$$Ht_A = 5\left(\frac{M_B}{10^2 M_\odot}\right)^{1/3}\left(\frac{10^{-12}\mathrm{G}}{B_0}\right)\left(\frac{\Omega_B h^2}{0.0125}\right)^{1/6}. \tag{3}$$

Bearing in mind the discussion below eq. (1), this means that the angular momentum removal timescale for an extragalactic $M_B \lesssim 10^2 M_\odot$ protostar is less than the Hubble timescale when there is a primordial magnetic field of present strength $B_0 \sim 10^{-12}$G, on a length scale relevant to the formation of an $M_B \lesssim 10^2 M_\odot$ primordial protostar. (Under appropriate conditions, molecular hydrogen cooling sufficiently lowers the Jeans mass, Palla, Salpeter, & Stahler 1983, also see Hogan 1993, so thermal-pressure support need not be significant, and for the cases discussed here magnetic-pressure support is not significant.) Given the uncertainties, and the weak dependence on M_B in eq. (3), it would not be surprising if a more careful numerical estimate found that primordial magnetic braking effectively removed angular momentum from even a collapsing $M_B \sim 10^{3-4} M_\odot$ cloud.

It is, however, unlikely that the magnitude of a primordial seed magnetic field thought reasonable will allow for the efficient removal of angular momentum from a collapsing $M_B \sim 10^8 M_\odot$ protoobject. Given that it is believed that the black hole responsible for a quasar resides in the nucleus of a virialized galactic halo, we propose instead a two-stage process to form quasar black holes of $M_B \sim 10^8 M_\odot$. Since galaxies (and larger structures) do not have an angular momentum problem, we first consider the collapse, at high redshift, of a host (a galactic halo) of baryonic mass $M_{B,H}$, and then the further collapse of a baryonic mass M_B nucleus within the halo (possibly prior to significant star formation). During the collapse of the host the primordial seed magnetic field will be amplified, and the resulting larger magnetic field in the host means that the angular momentum removal timescale for the central core of the host, of mass M_B, will be shorter than it is for a primordial object of the same mass, eq. (1). Thus the massive black hole needed for a quasar is more likely to be produced by the magnetic braking of the central core of the galactic halo host.

Flux conservation and the assumption of isotropic collapse relates the magnetic field B_H inside the collapsed host, with baryon density $\rho_{B,H}$, to the primordial field and baryon density, through $B_H = B(\rho_{B,H}/\rho_B)^{2/3}$. Further anisotropic collapse to a disc and/or differential rotation of the host would further amplify the magnetic field; for simplicity, we ignore both these effects in what follows. During the early stages of collapse of the central core of mass M_B, the field lines are twisted by the core spin-up and the resulting Alfvén waves carry angular momentum out to the outer parts of the host, and attempt to enforce corotation. Equation (1) applies, with $B = B_H$ and $\rho_B = \rho_{B,H}$. For times longer than t_A the angular momentum of the core is transferred to a mass bigger than the core and the change in the angular velocity of the host halo is small. A simple calculation shows that the efficiency of generation of the toroidal magnetic field increases as the core decreases in radius so that the rate of collapse increases rapidly as the core shrinks. Therefore, the time for total collapse is only a little longer than the time for an initial collapse of the core by a factor of two, which is essentially that given by the modified eq. (1). To be able to 'soak-up' the angular momentum being transferred, the host needs to be significantly more massive than the collapsing core, and a mass $M_{B,H} \sim 10^2 M_B$ is probably reasonable.

Of course, the greater density in the host leads to a faster dynamical timescale, but the relevant timescale on which angular momentum needs to be reduced is that of the age of the universe at the given epoch, which we will again approximate by H^{-1}. Once again ignoring the density difference between the core and the rest of the host during the early stages of collapse of the core, and considering spherical collapse with parallel magnetic field and angular momentum, we find, for a core $M_B = 4\pi\rho_{B,H}R^3/3$,

$$Ht_A = \frac{8}{15}(36\pi)^{1/6}\frac{H_0}{B_0}\frac{(\rho_0\Omega_B)^{2/3}}{\sqrt{\rho_{B,H}}}M_B{}^{1/3}\left[\Omega_0 + \frac{\Omega_\kappa}{1+z} + \frac{\Omega_\Lambda}{(1+z)^3}\right]^{1/2}(1+z)^{3/2}, \qquad (4)$$

or numerically,

$$Ht_A = 4\left(\frac{M_B}{10^8 M_\odot}\right)^{1/3}\left(\frac{10^{-12}\mathrm{G}}{B_0}\right)\left(\frac{\Omega_B h^2}{0.0125}\right)^{2/3}\left(\frac{10^{-24}\mathrm{g\,cm^{-3}}}{\rho_{B,H}}\right)^{1/2}\left(\frac{1+z}{6}\right)^{3/2}, \qquad (5)$$

where we have again set $h[\Omega_0 + \Omega_\kappa/(1+z) + \Omega_\Lambda/(1+z)^3]^{1/2} = 0.3$, and this $\rho_{B,H}$ is conservative, since the density near the core will be higher. In this case, since Ht_A is no longer a ratio of the Alfvén and dynamical timescales (as in eq. [2]), it depends much more strongly on the redshift (eq. [4]), even at large redshift. Given the uncertainties, and bearing in mind the discussion above, eq. (5) shows that magnetic braking in a 'cosmological' magnetic field ($\sim 10^{-12}$G) amplified by the collapse of a host can efficiently remove the angular momentum of a $M_B \sim 10^8 M_\odot$ collapsing core.

3. DISCUSSION

The inconsistency of the kinematic mean-field galactic dynamo scenario (Kulsrud 1988, 1990a; Vainshtein & Rosner 1991; Cattaneo & Vainshtein 1991; Vainshtein & Cattaneo 1992; Kulsrud & Anderson 1992; reviewed in Ratra 1991b), has focussed attention on the possibility that large-scale galactic magnetic fields are remnants of a weaker pregalactic cosmological magnetic field that has been amplified by anisotropic protogalactic collapse and galactic differential rotation (Piddington 1970, 1981; Kulsrud 1986, 1988, 1990b; reviewed in Ratra 1991b). In this scenario one would expect an intergalactic magnetic field of present strength $\sim 10^{-12}$G on a scale of a few Mpc (Kulsrud 1988, 1990b).

Such a field can be generated in a variant of the early universe inflation picture (Ratra 1992b, 1991b). In this class of inflation models the energy density perturbation power spectrum also depends on the redshift of reheating, so the parameters of the inflaton field potential are not unduly constrained by observational data (Ratra 1992a, 1991a). Weaker fields can also be generated in other variants of the inflation scenario (e.g., Dolgov & Silk 1993; Garretson, Field, & Carroll 1992; Dolgov 1993), or during an early universe phase transition (e.g., Enqvist & Olesen 1994; Cheng & Olinto 1994), or from the evolution of topological defects (Dolgov & Rhie 1992; Sicotte 1994). Other possibilities include a field generated near black holes in galactic cores (Hoyle 1969; Daly & Loeb 1990; Chakrabarti, Rosner, & Vainshtein 1994), or in the first generation of galactic stars, and then spread through galaxies; in both these cases one cannot appeal to magnetic braking to allow formation of the compact objects, so it is unclear how such objects form.

Observations cannot yet rule out an intergalactic magnetic field $\sim 10^{-12}$G on a scale of a few Mpc at the present epoch (Kronberg 1994), although measurements of magnetic fields in higher-redshift objects (e.g., Perry, Watson, & Kronberg 1993) constrain the possibilities, and other proposed techniques (Chen 1995; Plaga 1995) might also help.

Our estimates of the angular momentum removal timescale, set by magnetic braking, is relevant during the initial 'low-density' stage of protoobject collapse. Effects associated with a disk or bar, fragmentation, internal gravitational tidal torques, dynamical friction, shocks, etc. (all of which we have assumed to be insignificant during the initial collapse stage) will influence the 'higher-density' stage of the collapse, and must be accounted for to determine when the final stage of collapse is a solitary compact object, or one surrounded by a disk, or a compact cluster of compact objects, etc. As we are interested here in the formation of compact objects, in what follows we focus on this case.

We have shown that if the present cosmological magnetic field is $\sim 10^{-12}$G on scales relevant to the collapse of primordial clouds of mass $M_B \lesssim 10^2 M_\odot$, magnetic braking effectively reduces the angular momentum barrier and allows the collapse of such protoobjects. This is in the mass range of extragalactic Pop III stars now under discussion (e.g., C94), and black holes of this mass are also of interest. (Since lower mass objects are more efficiently braked, primordial magnetic braking also allows for the formation of extragalactic brown dwarves and other low mass stars; under appropriate conditions, molecular hydrogen cooling can sufficiently lower the Jeans mass, Palla et al. 1983.) It is conceivable that a more careful analysis will show that primordial compact objects with $M_B \sim 10^{3-4} M_\odot$ could also be allowed to collapse via this mechanism. Pop III stars in this range are also under discussion, and if instead the final object is a black hole of $M_B \sim 10^4 M_\odot$, it would be near the lower end of what might act as a seed black hole that would grow by accretion, if embedded in a massive and dense collection of gas and stars, to eventually become the massive black hole thought necessary to describe the quasar phenomena (e.g., Hills 1975; Young, Shields, & Wheeler 1977; Duncan & Shapiro 1983; David, Durisen, & Cohn 1987; Murphy, Cohn, & Durisen 1991). It is now thought that the quasar phenomena is a short-lived epoch in nearly every reasonably sized galaxy (rather than a longer epoch in only large galaxies), and in this case the 'standard' scenario requires a more modest black hole $\lesssim 10^8 M_\odot$ in these more numerous galaxies (Sołtan 1982; Cavaliere & Padovani 1989; Padovani, Burg, & Edelson 1990; Chokshi & Turner 1992). We have shown that a primordial magnetic field of present strength $\sim 10^{-12}$G, amplified by collapse in a host of baryonic density $\rho_{B,H} \sim 10^{-24}$g cm^{-3} (presumably a galactic halo with $M_{B,H} \sim 10^{10} M_\odot$), can effectively remove the angular momentum of the central $M_B \lesssim 10^8 M_\odot$ core and so, under appropriate conditions, could permit the formation of $M_B \lesssim 10^8 M_\odot$ proto-black holes in hosts at $z \sim 5$ (in this case magnetic braking is even more effective at lower redshifts). As the angular momentum removal timescale (eq. [5]) depends on the host density and the redshift (among other parameters), we expect some correlation between the mass of the black hole and the density and time of formation of the host, as well as a spread in the black hole masses. (We again emphasize that lower mass cores will be braked more effectively. Magnetic braking could also help form globular clusters, if they are not primordial, but rather formed during or after the collapse of the host galaxy.) As the magnetic braking mechanism is quite effective at removing angular momentum, it is likely that proto-black holes (and quasars) can form in low-mass sub-L_* galaxies, which would be consistent with recent observations (Bahcall et al. 1994); the other possible interpretation, that these are really high-mass sub-L_* (because of delayed star formation) galaxies is less plausible, since it would seem unlikely that a process which led to the formation of a central black hole would not also result in significant star formation (furthermore, in some cases, the observed heavy element abundances would seem to require, at the very least, simultaneous star formation). Magnetic braking would seem to lead to the formation of moderately massive black holes in most reasonably sized galaxies, and this would likely require that the quasar phenomena in each such galaxy is short-lived, consistent with what is now thought reasonable (e.g., Padovani et al. 1990). A quantitative estimate of the distribution of black hole and host galaxy masses is needed before a firmer conclusion can be drawn.

Like in galactic protostellar formation, as the density builds up during the collapse, ambipolar diffusion eventually becomes significant and the magnetic field in the collapsing object does not build up as fast as a naive application of the isotropic collapse formula suggests (Mestel & Spitzer 1956). It is necessary to account for this (as well as reconnection) to be able to estimate the final magnetic field in the compact object. Magnetic fields may play a prominent role in phenomena like bipolar flows, jets, and accretion disks, that are associated with AGN (and probably also with primordial stars). The scenario discussed here might explain where these fields come from. It would be of interest to quantify these speculations, and in particular to estimate the resulting magnetic viscosity implied in the associated accretion disk (in those cases where one forms), which might, among other things, help remove the angular momentum from the fuel in the disk and so allow it to fall towards the central compact object and cause the quasar to 'shine'.

Although the model we use for our analytic estimate of magnetic braking timescales is idealized, we believe that 'cosmological' magnetic braking affords good prospects for effectively removing angular momentum and so allowing formation of extragalactic stars and the black holes needed in the 'standard' quasar scenario. Clearly, a more careful analysis is called for, possibly a numerical simulation of protoobject collapse in the presence of a primordial magnetic field, along the lines of Basu & Mouschovias (1994), but generalized to account for the expansion of the universe and angular momentum generated by tidal torques, preferably in a low-density open or cosmological-constant-dominated cosmological model (Ratra & Peebles 1994; Górski et al. 1995; Bunn & Sugiyama 1995; Stompor, Górski, & Banday 1995) which are now thought to be fairly reasonable descriptions of the universe.

We thank J. Peebles and E. Turner for helpful discussions. This work was supported in part by NSF grants PHY89-21378 and AST91-21847, by the David and Lucile Packard Foundation, by NASA grant NAGW-2419, and by a Columbus Fellowship.

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