GRP-245/CALT-68-1712
January 1991

Is Inflation Observationally "Fine-Tuned"?

Bharat Ratra
Theoretical Astrophysics and Theoretical Physics,
California Institute of Technology, Pasadena, CA 91125

The small spatial inhomogeneities that are assumed as high redshift initial conditions in the gravitational instability scenario for the formation of cosmological structure might be the remnants of quantum fluctuations during a very early epoch of inflation (a related proposal may be found in Ref. [1] and an early computation is described in Ref. [2]).

One of the first exponential expansion inflation models in which this mechanism was studied is the quartic potential scalar field dominated model, with the scalar field effective potential

$$V(\Phi) = V_0 - \frac{1}{4}\lambda\Phi^4 \tag{1}$$

where V_0 and λ are constant parameters. In this model it is found that the observational upper bound on the late time, large-scale, adiabatic energy density irregularity power spectrum strongly constrains the value of the coupling constant, $\lambda \lesssim 10^{-10}$, [3-6] — i.e., the model needs to be "fine-tuned" to not disagree with large-scale observational data. Over the last half a dozen or so years this result has motivated the search for inflation models that are not observationally "fine-tuned" (some of which might seem to be quite contrived).

We have analyzed the generation and evolution of spatial irregularities in a number of simple scalar field dominated inflation modified hot big bang models, [7-10]. We find that large-scale observational data does not unduly constrain the parameters of these models; in particular, the parameter that plays the role of the coupling constant in these models does not have to be exceedingly small. Given the discussion of the previous paragraph, this result is somewhat surprising, so we briefly summarize why we believe that it is premature to claim that the inflation scenario of the very early universe is observationally "fine-tuned" (a more complete discussion may be found in Ref. [11]).

The reason for the difference in the conclusions of Refs. [3-6] and of Ref. [8] (where the quartic potential inflation model is studied) is the difference in the prescriptions used to implement "reheating", [11]. In the earlier analyses, the approximate expression for the potential during inflation, eqn. (1), is used to determine when inflation ends (this is taken to be the time when the "slow-rolling" approximation breaks down). In our analysis eqn. (1) only holds during inflation and does not determine when inflation ends (which is when the energy density drops to an appropriate value). With the "reheating" prescription of Refs. [3-6] the scalar field takes a time of the order of the Hubble time (at this epoch) to change from the value it has at the end of inflation to the value it has at the minimum of the complete potential — this is the slow "reheating" prescription. With our "reheating" prescription the scalar field reaches the minimum of the potential in a time much shorter than the Hubble time — this is the rapid "reheating" prescription. Using the rapid "reheating" prescription one finds that the quartic potential inflation model cannot be unduly constrained by large-scale observational data, [8,11], in part, because the late time power spectrum depends on both λ and the value of the Hubble parameter at "reheating" (as well as one other free parameter, [8]). With the slow "reheating" prescription the power spectrum only depends logarithmically on the value of the Hubble parameter at "reheating", so the one parameter that it is quite sensitive to, λ, is strongly constrained by the observations.

Only in the case of the quartic potential inflation model does the slow "reheating" prescription result in a late time power spectrum that is not very sensitive to the value of the Hubble parameter at "reheating". If one considers an effective potential, [10],

$$V(\Phi) = M^4 - \frac{1}{4}\lambda M^{4-\alpha}\Phi^\alpha \tag{2}$$

where M is the energy scale of exponential expansion inflation and α is a parameter, one finds that the $\alpha = 4$ case is nongeneric (in that the dependence of the late time power spectrum on M is only logarithmic). More specifically, the dominant λ and M dependence of the late time baryon dominated epoch power spectrum of large-scale energy density irregularities is, [10],

$$(\lambda)^{2/(\alpha-2)} \left(\frac{M}{m_p}\right)^{2(\alpha-4)/(\alpha-2)}, \tag{3}$$

where m_p is the Planck mass. (In this model the slow "reheating" prescription may be used only if $\alpha > 2$, [10].) If $\alpha \neq 4$, and not too large, the power spectrum is roughly equally sensitive to λ and M, while the dominant M dependence drops out when $\alpha = 4$. Also if α is sufficiently large the late time power spectrum is almost independent of λ, so observational data cannot require that λ be extremely small (for sufficiently large α the slow "reheating" prescription is somewhat similar to the rapid "reheating" prescription, [10]). If one wishes to have a λ that is not very much smaller than unity (e.g., $\lambda \gtrsim 10^{-2}$) then α needs to be moderately large (e.g., $\alpha \sim 8$) and M needs to be somewhat smaller than is usually considered (e.g., $M \gtrsim 10^{10}$ GeV), [10].

In conclusion we emphasize that it is premature to claim that the inflation scenario of the very early universe is observationally "fine-tuned". We should, however, also emphasize that neither of the "reheating" prescriptions used have been shown to follow from the microphysics of "reheating", [11].

I acknowledge useful discussions with J. Preskill. This work was supported in part by the NSF, grant AST84-51725, by the DOE, contract DE-AC03-81ER40050, and by the California Institute of Technology.

References

1. K. Sato, Mon. Not. R. Astr. Soc. **195**, 467 (1981).

2. V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981).

3. S. W. Hawking, Phys. Lett. **115B**, 295 (1982).

4. A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).

5. A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).

6. J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. **D28**, 679 (1983).

7. B. Ratra, Princeton preprint PUPT-1102B (1989).

8. B. Ratra, Caltech preprint GRP-217/CALT-68-1594 (1989).

9. B. Ratra, Caltech preprint GRP-229/CALT-68-1666 (1990).

10. B. Ratra, Caltech preprint GRP-243/CALT-68-1702 (1990).

11. B. Ratra, Caltech preprint GRP-242/CALT-68-1701 (1990).