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Comments on "Dependence of Density Perturbations on the Coupling Constant in a Simple Model of Inflation"

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ABSTRACT

With the original assumptions and approximations, the quartic potential scalar field (Φ) dominated inflation model, with potential $V(\Phi) = M^4 - \lambda\Phi^4/4$, leads to a late-time large-scale energy density perturbation power spectrum $P \propto \lambda$. As a result the observational upper bound on P implies an upper bound on λ (which is required to be exceedingly small). We have found that an alternate set of assumptions and approximations leads to $P \propto \lambda^{-2}$ and, through the observational constraint, to a lower bound on λ. Falk, Rangarajan and Srednicki (hereafter FRS) have argued that $P \propto \lambda^{-2}$ is possible, but that the observations still lead to an upper bound on λ. We reconcile this by noting that the assumptions and approximations of FRS are closer, but not identical, to the original set and differ from ours, and that the simplest interpretation of their result is that $P \propto \lambda$, with a numerical prefactor that differs from that found in the original computation. In short, by using a different set of approximations FRS have found yet another (correct) expression for the energy density perturbation power spectrum in the quartic potential model.

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By explicitly deriving the expression for the late-time large-scale fractional energy density perturbation power spectrum P in a number of inflation models, we have found that P is significantly model dependent, and, in particular, depends on the form of the potential used to model 'reheating' and the end of inflation [1-4] (for a review see Ref. [5]). More specifically, in the quartic potential model with scalar field (Φ) potential

$$V(\Phi) = M^4 - \frac{1}{4}\,\lambda\Phi^4 \tag{1}$$

(where M is the mass scale of inflation and λ is the quartic coupling constant), the original slow-reheating prescription leads to $P \propto \lambda$, [6,5], while we have found that the rapid-reheating assumption (along with a perturbation expansion in λ) implies $P \propto \lambda^{-2}$, [2,5] (see eqn. (8) below).

In an interesting preprint, [7], Falk, Rangarajan and Srednicki (hereafter FRS) have argued that in the model with potential (1) it is possible that $P \propto \lambda^{-2}$ (consistent with the result of Ref. [2]), given the appropriate assumptions and approximations, but that consistency with the observations imposes a stringent upper bound on λ (consistent with, and stronger than, that found in Refs. [6]). We reconcile these conclusions by noting that the more interesting, and simplest, interpretation of the result of FRS is that $P \propto \lambda$ (consistent with the result of Refs. [6]; see eqn. (5) below), but with a significantly different numerical prefactor than is found in Refs. [6]. That is, FRS have found a third (correct) expression for P in the inflation model which is partially described by the scalar field potential in eqn. (1). This result highlights the fact that the specification of the form of the inflation epoch scalar field potential is not a complete specification of the inflation model.

Using the techniques of Ref. [4], in the inflation model with the potential of eqn. (1), FRS find for the homogeneous scalar field during inflation, in the slow-roll approximation

$$\Phi(t) = \left[\frac{1}{(\Phi_*)^2} + \frac{2}{3}\,\frac{\lambda}{H}(t_* - t)\right]^{-1/2} \tag{2}$$

[their eqn. (4)] where t_* is the time when inflation ends, $\Phi_* = \Phi(t_*)$, and H is the Hubble parameter during inflation. (We emphasize that the analysis of Ref. [2] does not make use of the slow-roll approximation.) They note that when the second term in the square parentheses in eqn. (2) dominates (at first Hubble radius crossing) one recovers the result of the original computation, [6], for the fractional energy density perturbation at second Hubble radius crossing $\delta_{\rm ORG}$ [their eqn. (9)]. They argue that when the first term dominates they recover the result of Ref. [2] $\delta_{\rm ALT}$ [their eqn. (10)].

It is straightforward to establish that the expressions of FRS satisfy

$$\delta_{\rm ALT} = \delta_{\rm ORG} \left(\frac{{\Phi_{\rm SR}}^2}{2\,\Delta\beta\,{\Phi_*}^2} \right)^{3/2} \tag{3}$$

where $\Delta\beta = H(t_* - t_c)$ (t_c is the time of first Hubble radius crossing during inflation) is the number of e-foldings between first Hubble radius crossing and the end of inflation, and $\Phi_{\rm SR} = H\sqrt{3/\lambda}$ is the value of the field when $\ddot{\Phi} = 3H\dot{\Phi}$ [with $\Phi(t)$ given by eqn. (2)] (i.e., it is the value of the field when the slow-roll approximation used to derive eqn. (2) breaks down).

In Ref. [4] it was assumed that $\Phi_* = \Phi_{\rm SR}$; FRS generalize this to $\Phi_* = 10^{-\nu}\Phi_{\rm SR}/\sqrt{\Delta\beta}$ where ν is a numerical factor [see the discussion in the paragraph following their eqn. (10)] so the second factor on the right hand side of eqn. (3) is

$$\left(\frac{{\Phi_{\rm SR}}^2}{2\,\Delta\beta\,{\Phi_*}^2} \right)^{3/2} = \frac{10^{3\nu}}{\sqrt{8}}. \tag{4}$$

As a result

$$\delta_{\rm ALT} = \frac{10^{3\nu}}{\sqrt{8}}\,\delta_{\rm ORG}, \tag{5}$$

that is, the λ dependence of $\delta_{\rm ALT}$ and $\delta_{\rm ORG}$ are identical, [8], and so the conclusion of FRS must be that the observational upper bound on δ places an upper bound on λ, regardless of whether one uses $\delta_{\rm ORG}$ or $\delta_{\rm ALT}$. This is in agreement with the conclusion of Refs. [6], but in contrast to the conclusion of Ref. [2] where, since $\delta \propto \lambda^{-1}$ as a consequence of a different set of assumptions and approximations, a lower bound on λ follows.

We finally sketch the reason for the λ^{-1} dependence of δ in the rapid-reheating picture (for details see Ref. [2]). The equation of motion for the homogeneous background scalar field during inflation is

$$\ddot{\Phi} + 3H\dot{\Phi} - \lambda\Phi^3 = 0 \tag{6}$$

(eqn. (3) of Ref. [7]). With an appropriate choice of initial conditions the linear perturbation (in λ) solution of this equation is

$$\Phi(t) = \Phi_{0i} + \frac{\lambda}{3H}\,{\Phi_{0i}}^3(t - t_i), \tag{7}$$

where Φ_{0i} is the (time-independent) initial value of the order λ^0 part of the homogenous scalar field at $t = t_i$. From eqn. (7) we have

$$\delta \propto \frac{1}{\dot{\Phi}(t_c)} \propto \frac{1}{\lambda{\Phi_{0i}}^3} \tag{8}$$

(the first relation is not necessarily quantitatively accurate, [5]), so to get a numerical value for δ in the rapid-reheating picture one needs a model for what happens 'before' inflation to estimate Φ_{0i}. (The expansion in eqn. (7) need not be truncated at order λ^1; however, without a numerical value for Φ_{0i} it would not be a very useful exercise to work to higher order in λ.)

In conclusion we emphasize that the explicit form of the inflation epoch scalar field potential is not a complete specification of the inflation model.

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7. T. Falk, R. Rangarajan and M. Srednicki, Santa Barbara preprint UCSBTH-92-20, Phys. Rev. **D** (in the press) (1992).

8. M. Srednicki (email communication, 1992) has suggested that it might be possible for the parameter ν to be λ dependent, in a manner that ensures $10^{3\nu} \propto \lambda^{-3/2}$ and so $\delta_{\rm ALT} \propto \lambda^{-1}$ (as found in Ref. [2]). This possibility deserves further investigation; however it is not what is assumed in Ref. [7].