GRP-230/CALT-68-1667
August 1990

Effect of “Reheating” on Inflation Epoch Spatial Irregularities

Bharat Ratra

Theoretical Astrophysics and Theoretical Physics,
California Institute of Technology, Pasadena, CA 91125

ABSTRACT

Using a massive scalar field to phenomenologically model the large-scale effects of “reheating”, we show, in a spatially flat, scalar field dominated, cosmological model which inflates, that the amplitude of the late time large-scale fractional energy density irregularity power spectrum increases as the slope of the inflation epoch scalar field potential is decreased. This result corroborates our earlier conclusion that it is premature to claim that the inflation scenario of the very early universe has a “fine-tuning” problem.

Submitted to *Physics Letters B*

One of the conclusions of our recent analyses of the generation and evolution of spatial irregularities in simple inflation modified hot big bang cosmological models was that observational data does not unduly constrain the parameters of the inflation epoch scalar field potential, [1-4]. This result is at variance with the conclusions of almost all previous analyses of the de Sitter (exponential expansion) inflation model and our suggested explanation for this discrepancy was that previous analyses were incomplete, [1,3,4].

Given the simplified models studied, and the approximate techniques used, in Refs. [1-4] (which were needed to achieve the desired analytic understanding of the generation and evolution of spatial irregularities and justified for a preliminary analysis of this problem), we must also consider the alternate possibility that this discrepancy might be the result of an oversimplified treatment of "reheating" in the models of Ref. [1,3,4]. To examine this possibility we have constructed a simple, large-scale phenomenological, model of "reheating", [5]. We consider a spatially flat cosmological model dominated by a scalar field (Φ) with the scalar field potential, [5],

$$V(\Phi)\ \begin{cases} \propto\ \exp\bigl(-\Phi/\sqrt{p}\bigr) & \text{for } \Phi < \Phi^{(1)} \\ = m^2\left(\Phi - \Phi^{(2)}\right)^2 & \text{for } \Phi^{(1)} < \Phi < \Phi^{(2)} \\ = 0 & \text{for } \Phi^{(2)} < \Phi; \end{cases} \tag{1}$$

with the parameter $p > 1$ the first epoch describes an inflation model (Refs. [6,7,1]), $\Phi^{(1)}$ denotes the value of the scalar field at the transition to the second ("reheating") epoch (where m is the scalar field mass) and $\Phi^{(2)}$ is the value of the scalar field at the second transition to the scalar field kinetic energy dominated epoch. The parameters of the potential are chosen to ensure that the parts of the potential line up at the transitions (derivatives of the parts of the potential need not do so), [5].

Although the latter part of this cosmological model is probably not realized in the real universe, its advantages, for the purpose of settling an issue of principle, outweigh this liability; these include the fact that this model provides a more complete description of the large-scale effects of "reheating" (than those of Refs. [1,3,4]) and, the transitions here are simpler than in the models of Refs. [1,3,4], since they are between different scalar field dominated epochs (rather than, for instance, between a scalar field dominated inflation epoch and a radiation dominated epoch). We emphasize that the model does not account for the small-scale microphysics of the "reheating" transition and is only used to examine the large-scale effects of this transition on the evolution of inflation generated spatial irregularities.

The general procedure used to analyze this model is similar to that used in Refs. [1,3,4]. Using the techniques of Ref. [8] we solve the lowest and first order relativistic linear perturbation equations of motion (the Einstein equations and the scalar field equation linearized about a spatially homogeneous scalar field and FLRW cosmological model) in each of the three epochs: during inflation in Ref. [1]; during "reheating" in Ref. [9] (where we average over the fast scalar field variation, on a time scale of order m^{-1}, in the Einstein equations and make use of an expansion in m^{-1}, which is valid provided the redshift of "reheating" is not too large and provided "reheating" occurs sufficiently rapidly, [9,5]); and, during kinetic energy dominance in Ref. [5]. The constants of integration in the inflation epoch solutions are determined, from the quantum mechanically motivated initial conditions of Refs. [10,11,1], in Ref. [1], while those in the "reheating" and kinetic energy dominated epoch solutions must be determined from joining conditions at the transitions, [1].

In Refs. [1,3,4] the transitions were approximated as instantaneous, i.e., we required that the equation of state (and the pressure) change instantaneously and discontinuously at the transition spatial hypersurface, which was chosen to be a spatially homogeneous local (total) energy density spatial hypersurface. Joining conditions were determined by requiring that the equations of motion be singularity-free at the transition spatial hypersurface. (The joining conditions of Ref. [1] differ from those found in the literature; a restricted set of these joining conditions, the gauge-invariant joining conditions for ideal fluid dominance to ideal fluid dominance transitions, have been confirmed in Ref. [12].) Here we again approximate the transitions as instantaneous, however we now replace a pretransition scalar field potential by a posttransition scalar field potential at a spatially homogeneous local (total) scalar field spatial hypersurface, [5]. Since the parts of the potential have been chosen to match at the transitions, we find that these joining conditions result in the spatially homogeneous parts of the scalar field energy density and scalar field pressure not jumping at the transitions — the transitions in the model of Ref. [5] are gentler than those in the models of Refs. [1,3,4]. Also, since the two macrophysical prescriptions for "reheating" (that of Ref. [1] and that of Ref. [5]) as well as the two sets of joining conditions at the transitions (those of Ref. [1] and those of Ref. [5]) are physically distinct, we might hope to be able to isolate those late time characteristics of the inflation epoch model that would seem to not depend on the details of the "reheating" model or on the joining conditions.

These joining conditions are used to determine the constants of integration in the "reheating" and kinetic energy dominated epoch solutions. Since this cosmological model is not meant to account for small-scale (subhorizon scale) microphysics and since the most straightforward, observationally accessible, characterization of the inflation epoch model is provided by large-scale perturbations (i.e., those spatial momentum modes that were outside the horizon during the transitions), we restrict our expressions to large-scale irregularities.

The first result of interest is that the spatial momentum dependence of the large-scale two-point correlation functions of gauge-invariant fractional energy density irregularities (the adiabatic power spectrum) during "reheating" and during kinetic energy dominance are identical (as is that of the two-point correlation functions of gravitational wave perturbations), [5], and coincide with that of these statistics during baryon dominance in the model of Ref. [1] (where inflation is modelled in the same manner as in Ref. [5]). This result leads us to conclude that the spatial momentum dependence of these late time statistics is fixed by the inflation epoch model (and does not depend on the manner in which the late time universe is modelled).

The second result of interest is that the late time large-scale adiabatic power spectrum "diverges" in the de Sitter inflation limit. We find, during "reheating", that, in this limit, the large-scale adiabatic power spectrum is given by, [5],

$$P_{\Delta(R)}(k,t) = \frac{1}{\epsilon^2}\,\frac{16\pi}{m_p^2}\,k\,|\hat{A}_R|^2\,(a_R)^2, \tag{2}$$

where k is spatial coordinate momentum, ϵ $(= 1/\sqrt{p})$ is the slope of the de Sitter inflation epoch scalar field potential ($\epsilon \propto V'(\Phi)$ and $\epsilon \to 0$ is the de Sitter inflation limit), $m_p = G^{-1/2}$ is the Planck mass (where G is the Newtonian gravitational constant), $|\hat{A}_R|$ is an ϵ-independent factor (defined in eqn. (6.34) of Ref. [5]) and a_R is the scale factor during "reheating". It is pleasing to note that the late time power spectrum is of the scale-invariant form ($\propto k$), [13], as is expected of de Sitter inflation, [11,1,3,4], and that the power spectrum temporal dependence $[\propto (a_R)^2]$ is what is expected in the massive scalar field model (the temporal dependence has to be the same as that of the power spectrum in the pressureless ideal fluid model, [5,9]). In the de Sitter inflation limit, the kinetic energy dominated epoch large-scale adiabatic power spectrum is, [5],

$$P_{\Delta(K)}(k,t) = \frac{1}{\epsilon^2}\,\frac{16\pi}{m_p^2}\,k\,|\hat{A}_K|^2\,(a_K)^8, \tag{3}$$

where the ϵ-independent factor $|\hat{A}_K|$ is defined in eqn. (6.38) of Ref. [5] and a_K is the scale factor during kinetic energy dominance. Again, the power spectrum is of the scale-invariant form, while the rapid temporal growth [$\propto (a_K)^8$] is due to the stiff equation of state during kinetic energy dominance (the scalar field model in this epoch corresponds to a fluid model with background equation of state $p_b = \rho_b$, where p_b and ρ_b are the background pressure and energy density, and effective "speed of sound" $c_{s\Phi(K)} = 1$), [5].

As expected, the spatial momentum dependence of eqns. (2) and (3) are identical and agree with that of the de Sitter inflation limit power spectra in the baryon dominated epoch of the model of Refs. [1,3] and of the model of Ref. [4]. It is interesting to note that the numerical prefactors in all of these large-scale late time power spectra "diverge" in this limit; in fact, the "divergences" here and in the baryon dominated epoch of the model of Refs. [1,3] (which also makes use of the exponential potential scalar field inflation model) are identical — the "divergence" is determined by the inflation epoch model and seems to be independent of how the late time universe is modelled.

The reason for the de Sitter inflation limit "divergence" of the late time power spectrum here is similar to that responsible for the "divergence" in the model of Refs. [1,3] and in the model of Ref. [4]. In the de Sitter limit the inflation model reduces to a time-independent cosmological constant model and the background scalar field (and energy density) evolves exceedingly slowly. This causes a prominent warping of the spatially homogeneous scalar field hypersurfaces (as well as the spatially homogeneous energy density hypersurfaces) relative to the synchronous gauge constant time hypersurfaces, [5,1,3,4] (however, neither is the inflation epoch scalar field irregularity nor is the inflation epoch energy density perturbation "divergent" in the de Sitter inflation limit, [5,1]). At the transitions we join the spatial irregularities on spatially homogeneous scalar field spatial hypersurfaces (in this model, and on spatially homogeneous energy density spatial hypersurfaces in the models of Refs. [1,3,4]) so the hypersurface warp is carried through to late times. It may be shown that these late time warped hypersurfaces imply that the late time power spectra "diverge" (this is easily accomplished for the models of Refs. [1,3,4], but is somewhat less straightforward to establish for the model studied here, [5]). (We note that, in the de Sitter inflation limit, the scalar field irregularity and the energy density inhomogeneity "diverge" during "reheating" and kinetic energy dominance, so the standard assumption of linear perturbation theory is not valid, during these epochs, for exceedingly small ϵ.)

In short, the "slowly rolling" background scalar field during de Sitter inflation is responsible for the "divergence" in the late time large-scale energy density power spectrum.

Since the de Sitter inflation limit late time power spectra "diverge" as ϵ^{-2} (in this model and in the models of Refs. [1,3,4]), observational data cannot provide an upper limit on ϵ (they do however provide a lower limit, [1,3]) and so the widely held belief that the observations require ϵ to be exceedingly small (the so-called "fine-tuning" problem of the inflation scenario) would seem to be without basis.

We emphasize that while the "reheating" prescription used here is phenomenologically more complete than that used in Refs. [1,3,4], it also has not been shown to follow from microphysical considerations (this is true of all "reheating" prescriptions considered to date). Nevertheless, it is gratifying that the main features of the large-scale late time statistics (excluding their temporal dependence) seem to be independent of both the "reheating" prescription as well as the manner in which the late time universe is modelled and so would seem to be universal, determined solely by the model of the inflation epoch.

In conclusion we underscore that it is, at the very least, premature to claim that the inflation scenario of the very early universe has a "fine-tuning" problem.

I am indebted to J. Peebles and L. Susskind for valuable advice. This work was supported in part by the NSF, grant AST84-51725, by the DOE, contract DE-AC03-81ER40050, and by the California Institute of Technology.

References

1. B. Ratra, Princeton preprint PUPT-1102 (1989).
2. B. Ratra, Phys. Rev. **D40**, 3939 (1989).
3. B. Ratra, Princeton preprint PUPT-1132 (1989).
4. B. Ratra, Caltech preprint GRP-217/CALT-68-1594 (1989).
5. B. Ratra, Caltech preprint GRP-229/CALT-68-1666 (1990).
6. F. Lucchin and S. Matarrese, Phys. Rev. **D32**, 1316 (1985).
7. B. Ratra and P. J. E. Peebles, Phys. Rev. **D37**, 3406 (1988).
8. B. Ratra, Phys. Rev. **D38**, 2399 (1988).
9. B. Ratra, Caltech preprint GRP-228/CALT-68-1665 (1990).
10. B. Ratra, Phys. Rev. **D31**, 1931 (1985).
11. W. Fischler, B. Ratra and L. Susskind, Nucl. Phys. **B259**, 730 (1985).
12. J.-c. Hwang and E. T. Vishniac, *Gauge-Invariant Junction Conditions for Cosmological Perturbations*, Austin preprint (1990).
13. E. R. Harrison, Phys. Rev. **D1**, 2726 (1970);
    P. J. E. Peebles and J. T. Yu, Ap. J. **162**, 815 (1970);
    Ya. B. Zel'dovich, Mon. Not. R. Astr. Soc. **160**, 1P (1972).