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CDM COSMOGONY IN AN OPEN UNIVERSE

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ABSTRACT

We summarize a computation of the cosmological inhomogeneities that are produced by quantum-mechanical fluctuations during an early epoch of inflation in a low-density cold dark matter model with open spatial sections. At $\Omega_0 \sim 0.3 - 0.4$ and $h = 0.65$, the results are observationally encouraging, and indicate that the low-density open model deserves more careful scrutiny.

1. Introduction

The COBE-DMR discovery[1], and the confirmation through a cross-correlation analysis with the FIRS data[2], of a large-scale cosmic background radiation (CBR) anisotropy a factor of $\gtrsim 2$ larger than what had been predicted in the scale-invariant, biased, cold dark matter (CDM) dominated, spatially-flat model (after normalizing to small-scale observational data at $\sim 8h^{-1}\mathrm{Mpc}$, where the present value of the Hubble parameter $H_0 = 100h\,\mathrm{km\,s^{-1}Mpc^{-1}}$), has been further strengthened by refined analysis of more COBE data[3], and by an analysis of the FIRS autocorrelation data[4].

While the details are still under debate, it seems reasonable to believe that this result will join the growing list of observational evidence that argues against a scale-invariant spatially-flat CDM model. This list of observations would include: dynamical mass estimates on length scales $\lesssim 10h^{-1}\mathrm{Mpc}$ which indicate[5] a low value for the clustered mass density parameter $\Omega_0 = 0.2 \pm 0.1$ (consistent with what is deduced[6] from rich cluster X-ray emission and standard nucleosynthesis); measurements[7] of the Hubble parameter which are consistent with $h > 0.6$ and globular cluster age estimates which suggest[8] an age of the universe $t_0 > 12\mathrm{Gyr}$; and measurements[9] of large-scale mass fluctuations which seem to prefer $\Omega_0 h \sim 0.1$ to 0.3 for a scale-invariant CDM spectrum. We emphasize that some of these are still preliminary observational suggestions, and that on scales $\gtrsim 20h^{-1}\mathrm{Mpc}$ the IRAS/POTENT analysis[10] suggests $\Omega_0 \approx 1$, although redshift surveys[11] on these and slightly smaller scales are also consistent with lower Ω_0.

The COBE CBR measurement could be consistent with the small-scale biasing needed in the $\Omega_0 = 1$ spatially-flat model, if the primordial spectrum was 'tilted'[12], or in the mixed dark matter (MDM) scenario[13]. For these models to remain viable, improved estimates of h and t_0 must result in a value for ht_0 that differs from what is now generally thought to be preferred. These models are consistent with the IRAS/POTENT result and with measurements[9] of large-scale mass fluctuations.

Since small-scale mass fluctuations in these models are suppressed (either by the tilting of the primordial spectrum, or by the streaming of the hot component), they form structure late[14]. Since gravity does not have much time to gather mass around recently formed structures like galaxies and clusters, it is conceivable that the mass distribution around such objects is not representative of the global distribution, and it is not inconceivable that it might not be that far from what is inferred from small-scale dynamics.

However, because of late structure formation, and the rapid evolution of structure in a high Ω_0 universe[15], the tilted CDM and MDM models would seem to have a problem explaining observational evidence[16] which suggests that giant galaxies have not significantly evolved since redshift $z \sim 1$. Also, since clusters form almost at the present epoch in these models, one would expect that regions at $z = 1$ in which galaxies are clustered[17] are extremely rare. There are also indications[18] that the MDM model has difficulty accounting for the damped Lyα systems seen[19] at $z = 2$ to 4 (which are thought to be young galaxies).

Lowering Ω_0 is an alternate way of accommodating both the large-scale CBR data and the small-scale clustering data. Isocurvature perturbations[20,21] in an open or cosmological constant (Λ)-dominated flat model are a possibility, as are topological defects in an open universe[22]. Adiabatic perturbations in a low-density open or Λ-dominated flat model could also reconcile the small-scale observations with the COBE and FIRS measurements.

Given the uncertainties, all these low-density models are consistent[23,9] with the dynamical mass estimates on small scales, with the range of ht_0 that is indicated by current observations, and, given an appropriate spectrum of density fluctuations, with the measurements of large-scale mass fluctuations. For these models to remain viable, refinements of the IRAS/POTENT analysis will need to find a lower Ω_0, more in line with the estimates from large-scale redshift surveys. Structure forms earlier in the low-density models, so they seem to be consistent with the current interpretation of high redshift observations.

Some people interpret the possible clash between the baryon density predicted by standard nucleosynthesis[24] and the mass density inferred from small-scale dynamical estimates as being indicative of the presence of CDM. In any case, in the adiabatic scenario it has proved difficult to find a purely baryonic model (i.e., excluding baryons bound up in compact objects which behave like CDM) that is consistent with the observations, so attention has focussed on low-density CDM models.

There is a version of the inflation scenario, in which a single-bubble open inflation model is created by a tunnelling event in a spatially-flat de Sitter spacetime which is also inflating[25]. In this model, the first, de Sitter, epoch of inflation smooths away preexisting irregularities. For the second epoch of inflation there are three sources of inhomogeneities, all quantum-mechanical in nature: fluctuations, created during the tunnelling event, in the initial spatial hypersurface of the bubble; fluctuations in the first epoch of inflation that eventually come into the bubble; and the more familiar ones generated on small scales inside the bubble. The first epoch of inflation is important, since it gets rid of most fluctuations, and in particular, those on the scale set by space curvature in the second epoch of inflation, which, if large, would result in unacceptable[26] large-scale CBR anisotropies. (Since the energy-density difference between the false and true vacuum is very small[25], the bubble walls are not very significant.) A variant of this open inflation scenario could accommodate a low-density open model[27,28].

In such an open inflation model, it is not implausible that the inhomogeneities generated by the first two mechanisms are subdominant. Standard techniques allow one to compute the spectrum of energy-density perturbations generated by quantum-mechanical fluctuations during the second epoch of inflation[29,30,27]. On large scales the form of the spectrum depends on both the Hubble length scale and that set by space curvature. For a relatively flat potential, this spectrum is the open model generalization of the familiar scale-invariant[31] spectrum of the flat model, and, because of the dependence on space curvature, is somewhat more complicated than the spectra used previously in open model computations of the CBR anisotropy[32].

In the next section we summarize the computation of the open inflation model spectrum, and mention some possibly observable consequences of such a spectrum, and in the last section we conclude by considering the possibility of observationally distinguishing between the low-density open and Λ-dominated flat models.

2. The Open Model

For ease of physical interpretation, the computation of energy-density perturbations generated by quantum-mechanical fluctuations during the second epoch of inflation is done in the standard synchronous gauge approach[33,34,35,30,27]. (We prefer to use this method rather than the gauge-invariant formalism[36].)

The model we consider is divided into three epochs: an early scalar field inflation epoch, and the more familiar late-time radiation and nonrelativistic matter epochs. During inflation the inflaton scalar field Φ potential is

$$V(\Phi) = 12\bar{h}^2[1 - \epsilon\Phi], \tag{1}$$

where the first term, $12\bar{h}^2$, drives the expansion during inflation, and the second term, $\propto \epsilon$, forces Φ 'down the hill'. At reheating, $V(\Phi)$ drops to zero, and the Φ energy density is converted to radiation energy density[37]. The equation of state is also assumed to change instantaneously at the radiation-matter transition[37].

We compute to linear order in the matter and metric perturbations about a spatially homogeneous open cosmological model, and during inflation we also use a perturbation expansion in ϵ, working to lowest nontrivial order[38]. The constants of integration in the general solutions during inflation are determined from quantum-mechanical initial conditions[39,30,27], and those in the radiation epoch and nonrelativistic matter epoch solutions are determined from joining conditions at the transitions[37,27].

For a preliminary comparison to large-scale structure data, it is convenient to transform theoretical expressions to the instantaneously Newtonian synchronous hypersurface on which the observations are made[35]. The fractional energy density perturbation (δ) power spectrum on this hypersurface, $\widehat{P}(A,\hat{t}_N) = |\hat{\delta}(A,\hat{t}_N)|^2$, is given by[27,28], at the present time (and neglecting the transfer function),

$$\frac{\epsilon^2}{(1+z_{\rm re})^4}\widehat{P}(A) = 2\pi\left(\frac{H_0}{m_p}\right)^2 \frac{\Omega_0}{1+z_{\rm eq}}\left(\frac{W_1}{c_1}\right)^2 \frac{(4+A^2)^2(e_1+A^2)^2}{A(1+A^2)(d_1+A^2)^2}, \tag{2}$$

where the radial coordinate wavenumber A $(0 < A < \infty)$ is related to the eigenvalue of the scalar spatial Laplacian, $-(A^2+1)$, $z_{\rm re}$ and $z_{\rm eq}$ are the redshifts of reheating and equal radiation and matter energy densities, $m_p = G^{-1/2}$ is the Planck mass, $d_1 = (8+\Omega_0)/[2(1-\Omega_0)]$, $e_1 = 1 + 3c_1/W_1$, and

$$\frac{W_1}{c_1} = \frac{2(1-\Omega_0)}{45{\Omega_0}^{1.5}(1+z_{\rm eq})^{2.5}} + \frac{1+2\Omega_0}{1-\Omega_0} + \frac{3\Omega_0}{(1-\Omega_0)^{1.5}}\ln\left\{\frac{1}{\sqrt{\Omega_0}} - \sqrt{\frac{1-\Omega_0}{\Omega_0}}\right\}. \tag{3}$$

At large wavenumber $\widehat{P}(A) \propto A$ (the usual scale-invariant spectrum[31]), while at small wavenumber $\widehat{P}(A) \propto A^{-1}$. This large-scale asymptotic behaviour of the power spectrum is mostly a consequence of space curvature.

The model is normalized to the COBE measurement of rms CBR temperature anisotropy at 10°, $\delta T(10^\circ) = 35(1\pm0.3)\mu$K (since this number must account[40] for the effects of monopole- and dipole-subtraction corrections and for using a gaussian rather than the real DMR beam shape, it is a little larger than the quoted COBE value[3]). The computation of δT in the model is done in the gauge-invariant formalism[41], which takes as input the gauge-invariant fractional energy density perturbation (Δ) power spectrum $P_\Delta(A,t) = |\Delta(A,t)|^2$. At the present epoch this is given by[27,42],

$$\frac{\epsilon^2}{(1+z_{\rm re})^4}P_\Delta(A) = 2\pi\left(\frac{H_0}{m_p}\right)^2 \frac{\Omega_0}{1+z_{\rm eq}}\left(\frac{W_1}{c_1}\right)^2 \frac{(4+A^2)^2}{A(1+A^2)}. \tag{4}$$

This has the same asymptotic behaviour (in A) as $\widehat{P}(A)$, Eq. (2), and the wavenumber dependence of $P_\Delta(A)$ is consistent[43] with what may be derived from the expressions given in Ref. 29.

Both $\widehat{P}(A)$ and $P_\Delta(A)$ are proportional to $1/\epsilon^2$, so an observational upper bound can only impose a lower bound on ϵ, the slope of the inflation epoch scalar field potential[35,38,44].

The CBR anisotropy in comoving coordinates, $\delta T/T = \sum_{l,m} a_l^m Y_l^m$, may be characterized by the multipole moments $C_l = \langle |a_l^m|^2\rangle$. The C_l have been evaluated to $l = 100$ in Ref. 45 and to $l = 2000$ in Ref. 42. The shape of the C_l at low l is determined by the damping of the spatial harmonics on scales comparable to that of space curvature, and the asymptotic $1/A$ behaviour of the power spectrum, which is also a consequence of space curvature.

Other interesting consequences of these two effects of space curvature are the second zero in the mass two-point correlation function at separation $\sim 1300h^{-1}$Mpc for the relevant range of Ω_0, and the strong damping of two-point correlation functions of observables at separations comparable to that of space curvature[27,28].

Depending on the 10° CBR anisotropy, when $\Omega_0 \sim 0.3 - 0.4$ and $h = 0.65$, the open model appears to do reasonably well at fitting a fair fraction of the current observations[28,42].

3. Discussion

Low-density open and Λ-dominated flat CDM models[23,9,28,42,46] provide a fairly reasonable explanation for a significant fraction of current observational data.

At the level of global parameters of the cosmological model, at fixed Ω_0 and t_0, a larger value of h favours the Λ-dominated case. For the versions under discussion now, the two low-density models have significantly different values of the global parameters, so better observational constraints on the values of Ω_0, h and t_0 are likely to prove quite useful.

With the COBE CBR normalization, and the same Ω_0, the Λ-dominated model has a somewhat larger rms fluctuation $\delta M/M$ in the mass in a sphere of radius $8h^{-1}$Mpc than does the open case. (At $\Omega_0 = 0.3$, the $P_\Delta(A) \propto A$ open case[32] has a $\delta M/M(8h^{-1}\text{Mpc})$ that is $\sim 25\%$ larger than the open model considered here.) With an appropriate choice of parameters, and given the uncertainties, (mildly) biased versions of both models are not inconsistent with most low redshift observations, including the large-scale matter distribution[9], and the cluster mass and correlation functions[47].

At the same value of Ω_0, the open model predicts a lower rate of gravitational lensing of quasars by moderate-redshift foreground galaxies than does the Λ-dominated case[48]. The interpretation of the gravitational lensing constraint is complicated by the fact that obscuration by dust in the lensing galaxies might be significant[49], and that in the flat model one predicts[50] a very different rate of lensing for the same present value of a constant Λ and a time-variable Λ.

The large-scale CBR anisotropies in the models are not completely dissimilar[45], although in both cases they do differ from that in the scale-invariant Einstein-de Sitter model, so it would be difficult to use just the large-scale CBR anisotropy data to distinguish between the two low-density models. For the range of values of Ω_0 now under discussion, the position of the small-scale peak (in $l(l+1)C_l$ as a function of l) is more sensitive to Ω_0 in the open case[51] than it is in the flat Λ-dominated model[46]. This might help to distinguish between the models, but the weaker dependence on Ω_B, h, and ionization history, and the current limited observational resolution in l, should be kept in mind. Eventually, with a better understanding of the systematics of the CBR anisotropy observations, and with improved statistics, a combination of different CBR measurements should be able to distinguish between the two low-density cases.

The COBE CBR normalized Λ-dominated flat CDM model is consistent with a somewhat smaller Ω_0 and a somewhat larger $\delta M/M(8h^{-1}\text{Mpc})$, compared to the corresponding open case. As a result, structure forms a little earlier in the Λ-dominated model compared to the open model, and much earlier in both low-density cases than in the MDM and tilted CDM flat cases. It is not totally unreasonable to believe that a significant amount of observational data which bears on this point will soon be available.

In summary, while it is more straightforward to observationally distinguish the low-density CDM models from the Einstein-de Sitter, MDM, and tilted CDM cases, distinguishing between the low-density models is much more challenging. It is quite unlikely that any single observation will conclusively accomplish this, but it is not at all unreasonable to remain optimistic, given the rapidly improving, and ever expanding range of observational tests of world models.

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