J. Phys. B: At. Mol. Opt. Phys. **23** (1990) L637–L640. Printed in the UK

LETTER TO THE EDITOR

Charge-state dependence of elastic scattering cross sections at large angles between electrons and multiply charged ions

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Received 7 August 1990

Abstract. The differential elastic scattering cross sections of electrons from F^{q+} are calculated using a potential scattering model where each fluorine ion is represented by a static potential. The ratio of the cross section for each charge state q to the Rutherford differential cross section for charge $q=9$ is larger than unity for large angles, in particular, for $\theta = 180°$. The calculations are used to interpret the recently reported anomalous q dependence of the binary encounter peak in F^{q+}–He, H_2 collisions. It is also shown that similar binary encounter peaks observed at smaller angles will show different q dependence.

In a recent letter Richard *et al* (1990) reported on the projectile charge state (q) dependence of binary encounter electron (BEe) production in F^{q+} collisions with He and H_2. The electrons were observed at zero degrees with respect to the beam direction, which corresponds to $\theta = 180°$ in the projectile rest frame. Using the impulse approximation (IA) (Brandt 1983), which considers the target electrons as 'quasi-free' when the projectile velocity is much larger than the characteristic velocity of the target electrons, it was possible (Lee *et al* 1990) to fully explain the experimental double differential cross sections (DDCS) for *bare* ions. However, the recent work (Richard *et al* 1990) with q less than the nuclear charge of the ion showed an 'anomalous' q-dependence using IA at $\theta = 180°$. Similar studies (Stolterfoht *et al* 1974, Toburen and Wilson 1979) at $\theta \ll 180°$ exhibited the expected behaviour—the effective charge deduced from the data using the Rutherford formula is less than the nuclear charge of the projectile. Since the theoretical BEe production cross section in the impulse approximation is simply given by the differential elastic scattering cross section convoluted with the target momentum distribution (Compton profile), it is instructive to investigate elastic differential cross sections (DCS) of electrons from ions with different charge states.

It is the purpose of this letter to report on the differential elastic scattering cross sections of electrons from F^{q+}, and to present a comparison of theory with the recent experimental data (Richard *et al* 1990).

The elastic scattering between an electron and a F^{q+} ion is treated within the potential scattering model. The spherical symmetric potential of the ion (q+) consists of the electron–nucleus interaction and the screening due to the electrons in the ion. This potential can be written as $-q/r + V_s(r)$ where the latter is a short-range potential. The scattering amplitude for such a potential can be expressed (Joachain 1975) as

$$f(\theta) = f_C(\theta) + f_s(\theta)$$

where $f_C(\theta)$ is the Coulomb scattering amplitude for charge q and

$$f_s(\theta) = (2ik)^{-1} \sum_{l=0}^{\infty} (2l+1) \exp(2i\sigma_l)(\exp(2i\delta_l) - 1) P_l(\cos\theta)$$

where σ_l is the Coulomb phaseshift (with charge q) and δ_l is the additional phaseshift characteristic of the short-range potential $V_s(r)$. In the equation above, P_l is the Legendre polynomial of order l, and $E = k^2$ is the scattering energy (in Ryd) of the electron. The differential cross section is given by

$$\frac{d\sigma}{d\Omega}(\theta) = |f(\theta)|^2.$$

To calculate the screening due to the electrons in F^{q+} ions, Hartree–Fock wavefunctions of the occupied orbitals were calculated from which the static potential for each q was obtained. Alternatively, a model potential reproducing the energies of the ground state and the first few excited states of each ion was used. The phaseshift δ_l for each l, due to the short-range potential, was then calculated using the variable phase method (Calogero 1967) or by directly comparing wavefunctions calculated with and without the short-range potential. For $q = 3$, 5 and 7, we found that the phaseshifts using the two different methods for the two different potentials are practically identical for each l. In figure 1 the phaseshifts δ_l against l for $E = 40$ Ryd are shown. Since the phaseshift due to the long-range Coulomb potential has been included in $f_C(\theta)$ exactly, the phaseshifts due to the short-range potential decrease rapidly with increasing l, as shown in figure 1.

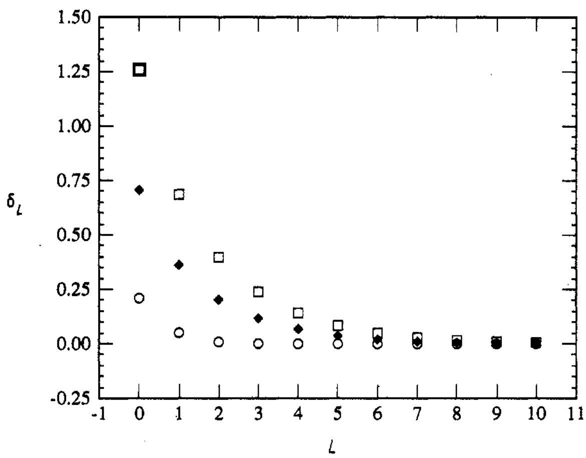

Figure 1. Calculated short-range phaseshifts δ_L (in rad) for the elastic $e^- - F^{q+}$ scattering at $E = 40$ Ryd: □, $q = 3$; ◆, $q = 5$; ○, $q = 7$.

The ratios of the differential cross sections for these ions to the corresponding Coulomb values for charge $q = 9$, $R(q, E, \theta)$, are plotted in figure 2 for $E = 40$ Ryd. It is striking to note that at large angles, and in particular at $\theta = 180°$, this ratio is *greater* than unity and increases with decreasing q. The ratio decreases for smaller scattering angles and approaches $(\frac{1}{9}q)^2$ as $\theta \to 0°$. From this figure, one also observes that the ratio R becomes less than unity and the q dependence is *not* monotonic if the

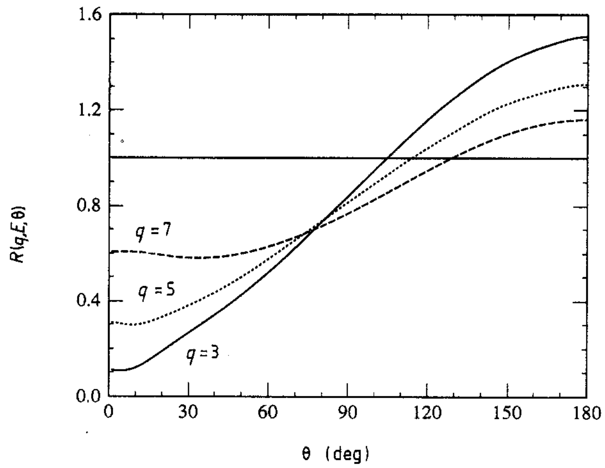

Figure 2. Variation of the calculated ratio of differential cross sections for $e^- + F^{q+}$ ($q = 3, 5, 7$) collisions with that for $e^- + F^{9+}$ collisions with scattering angles.

observation is made at smaller scattering angles. For small-angle scatterings ($\theta \ll 180°$), the differential cross sections do decrease with decreasing q. In figure 2, the flattening of each curve represents that the scattering can be approximated by the Rutherford scattering with an effective charge. For $q = 3$, this region is limited to a very small range of angles, reflecting that the short-range potential extends over a larger volume in comparison with higher q.

Next, we discuss the experimental results of BEe production (Richard *et al* 1990) in collisions of F^{q+} with He and H_2. Within the IA, the electron energies cover a range of approximately 22-90 Ryd. In figure 3 a plot of the ratio R at $\theta = 180°$ for this range of collision energy is given. The ratio is always larger for smaller q in this energy range and for $q = 5$ and 7 there is little energy dependence. Therefore a single multiplication factor to the Rutherford cross section for the bare charge $q = 9$ is sufficient to fit the data for $q = 5$ and 7, as was done in Richard *et al* (1990). The theoretical ratios from

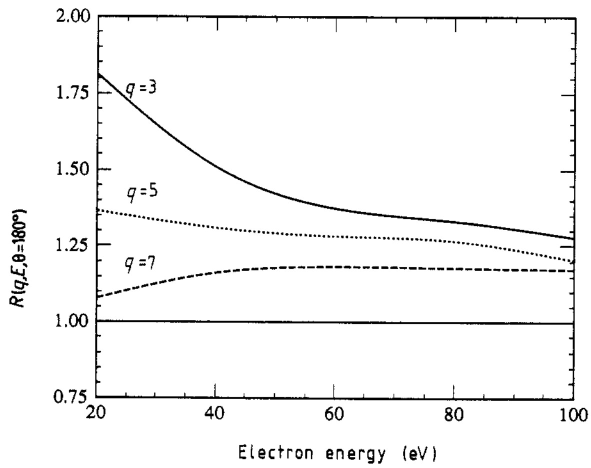

Figure 3. Variation of the calculated ratio of differential cross sections for $e^- + F^{q+}$ ($q = 3, 5, 7$) collisions with that for $e^- + F^{9+}$ collisions with the scattering energies of the electrons.

figure 3 for $q = 5$ and 7 are approximately 1.30 and 1.17, respectively. They are to be compared with the experimentally deduced values of 1.40 for $q = 5$ and 1.25 for $q = 7$. For $q = 3$, the ratio R varies with E significantly and the result in figure 3 has to be folded with the Compton profile in order to compare with the experimental BEe spectrum. The experimentally deduced ratio for 19 MeV F^{3+} on H_2, assuming that this ratio is independent of E, is about 1.50. This is consistent with the results in figure 3 since the BEe spectrum at this energy peaks at $E = 40$ Ryd where the ratio from figure 3 is about the same.

In conclusion, we have shown that the 'anomalous' q dependence in the binary encounter peaks in $F^{q+} + He$ and H_2 collisions can be understood in terms of the behaviour of the elastic scattering in $e^- + F^{q+}$ collisions at $\theta = 180°$. We have also shown that the q dependence of the binary encounter peak varies with the ejected angles of the electrons. Experimental confirmation of the predicted angular dependence is desirable which would also test the validity of the impulse approximation for such collisions.

This work at Kansas State University is partially supported by the US Department of Energy, Office of Basic Energy Research, Division of Chemical Sciences.

Note added in proof: The present results complement the recent work of Reinhold *et al* (1990).

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